A Stochastic Model for UAV Networks Positioned Above Demand Hotspots in Urban Environments

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Abstract—Wireless access points on Unmanned Aerial Vehicles (UAVs) are being considered for mobile service provisioning in commercial networks. These UAV access points will carry radio infrastructure and will be temporarily deployed in areas of dense user traffic to deal with excess user data demand. In this work, we analyse the coverage when UAVs act as access points for users on the ground in an urban area. We consider the impact of several UAV placement strategies, either independent of, or responsive to, the locations of the users on the ground. Our analysis allows us to demonstrate how the density of the UAVs in the network will determine whether the UAVs should position themselves closer to user hotspots to improve the received signal strength, or further away from one another to mitigate interference. Additionally, we demonstrate how network design parameters such as the UAV height above ground or the antenna beamwidth impact the coverage probability. In addition to simulations we provide mathematical expressions for the coverage probability, for the scenario where UAVs are positioned directly above the centers of user hotspots.

Keywords—UAV networks, coverage probability, poisson point process, stochastic geometry

I. INTRODUCTION

Radio infrastructure mounted on UAVs has become recognised by the wireless community as a key component of next-generation communication networks [1]. UAVs introduce several improvements over conventional infrastructure, such as the ability to intelligently adjust their positions in real-time and the ability to benefit from unobstructed wireless channels in the air. Due to these benefits, UAV-mounted infrastructure can be used for a range of applications, such as wireless sensor networks [2], public safety networks [3], and as flying small cells covering user hotspots in densely populated areas [4].

The introduction of UAVs also presents new challenges to network deployment, as the unprecedented flexibility of the new infrastructure requires more insight into how the new network parameters affect the achievable performance. As the UAVs can move in three dimensions on-demand, they can pursue a variety of deployment strategies with respect to the locations of end users, each other, or a combination thereof. The selection of the appropriate deployment requires an understanding of the performance impact of each deployment strategy in a given situation. In addition to this, the operating UAV height will affect the overall network performance and needs to be selected with care. Based on current regulations from the Federal Aviation Administration (FAA) [5] and the European Aviation Safety Agency (EASA) [6], as well as proposed airspace management schemes [7], we expect that UAVs serving urban hotspots will take the form of small, lightweight (below 25kg) devices operating at heights at or below 200m. Given this height range, the UAV network may be operating above a built-up urban area or below building heights in so-called urban canyons, which will significantly affect the radio environment experienced by the UAV network.

In this paper, we analyse the performance of a low-altitude UAV access point network which serves user hotspots in an urban environment. Using mathematical derivations as well as simulations we evaluate the network performance for different UAV placement strategies and comment on the impact of various network design parameters.

II. RELATED WORK & CONTRIBUTION

A. Related Work

The wireless community has published a number of works on the topic of deploying UAV-mounted access points to serve terrestrial users. These works typically set up an optimisation problem where a UAV parameter such as location in 3D space is optimised, subject to a given objective function. In [8] the authors consider a single fixed-wing UAV acting as a relay between two ground terminals, and the throughput is maximised by optimising the trajectory and the transmit power of the UAV across discrete timeslots. The same authors optimise the trajectory to maximise energy-efficiency in [9], and they optimise transmit power for a UAV with a circular orbiting pattern in [10]. In [11] the authors consider the problem of placing a UAV in 3D space to maximise the number of users that are covered, subject to a quality of service (QoS) constraint. In [12] the authors consider a UAV which dynamically repositions itself with respect to randomly moving users to maximise the achievable spectral efficiency at every discrete timeslot. In [13] the authors consider a scenario where a UAV relay iteratively searches an urban environment for locations where it can establish a Line-of-Sight (LOS) connection to the end user and Base Station (BS), meeting the channel rate requirements. In [14] the authors optimise the trajectory of a laser-powered UAV access point to maximise achievable throughput while meeting energy constraints. The authors propose a flight pattern which involves the UAV orbiting around the laser transmitter to harvest energy before hovering above the end user to maximise throughput. In [15] the author considers a scenario where each UAV in a network...
of UAVs iteratively adjusts its location to maximise its spectral efficiency, subject to interference from the remaining UAVs. In our previous work [16] we demonstrated how a K-means clustering algorithm can be used to position UAVs around user locations in an interference-free environment, and that the resulting UAV network outperforms fixed terrestrial networks in terms of received signal strength at the user locations. K-means clustering was also proposed for UAV placement optimisation in [17].

With the notable exception of [15], state of the art on UAV network optimisation tends to ignore the effects of interference, instead focusing on scenarios where individual UAVs are operating in isolation. Without interference the wireless links are limited by the geometry of the environment, and therefore the network performance is generally optimised through minimising the distance between the UAV and the receiver, as this minimises the pathloss, increases the LOS probability and enables the network to reduce transmit power. These optimisation strategies may not apply to a scenario where multiple UAVs are operating concurrently and creating interference for each other. In the presence of interference, decreasing the distances between transmitters and receivers may also have the result of decreasing the distances between interferers and receivers, potentially causing a net decrease in channel performance.

Stochastic geometry is an alternative method for modelling the spatial relationships in a UAV network in terms of long-term averages over many spatial realisations of deployments, capturing the effect of interference on network performance and giving us new insight into the performance trade-offs of UAV networks. In [18] and [19] the authors derive the coverage probability for a stochastic UAV network under guaranteed LOS conditions for fading-free and Nakagami-m fading channels. Stochastic geometry is applied by the authors of [20] to optimise UAV density in a radio spectrum sharing scenario under guaranteed LOS conditions. In [21] the authors evaluate the performance of a network of UAVs acting alongside a terrestrial BS network in an emergency outage scenario. The authors of [22] and [23] use stochastic geometry to evaluate the performance of a terrestrial BS network that is serving terrestrial users and UAV users simultaneously. In our previous works [24] and [25] we model the coverage probability of the user access and wireless backhaul links, respectively, of a UAV network operating in an urban environment in the presence of LOS-blocking buildings, assuming independent distributions of the transmitters and receivers.

B. Our Contribution

In our previous work [24] we investigated the coverage performance of a UAV network where the UAVs were positioned independently of user locations. In this paper we extend our work by considering a variety of additional UAV positioning strategies. These include UAV placement above user hotspot centers, optimised UAV placement using the K-means clustering algorithm [16], and UAV placement according to a rectangular grid.

Our contributions can be stated as follows:

1) We derive the coverage probability and average spectral efficiency experienced by a typical user for the case when UAVs are positioned directly above the centers of user hotspots. Our model takes into account parameters such as building density, user hotspot radius and UAV antenna beamwidth, and can represent different wireless fast-fading behaviours through Nakagami-m fading.

2) Using our model as well as simulations we demonstrate that there exists an optimum UAV height for a given user hotspot radius. We also demonstrate how the optimum UAV height is almost unaffected by varying the density of user hotspots and UAVs. Our numerical results demonstrate how interference imposes a strict limitation on the range of heights the UAV network can operate at.

3) We compare the performance of the UAV network deployed above user hotspots against a UAV network which is deployed randomly with respect to users, a UAV network which is deployed according to the K-means heuristic optimisation algorithm, as well as a UAV network deployed in a rectangular grid. This comparison allows us to demonstrate that for higher UAV altitudes and larger UAV densities the UAV network benefits more from UAVs positioning themselves with respect to each other rather than with respect to user hotspots, mitigating interference through minimising coverage overlap.

4) We additionally demonstrate that our derived mathematical expressions closely approximate the performance of a K-means algorithm that positions UAVs around user locations in a way which minimises the distances between the UAVs and their users.

III. System Model

A. UAV Positioning

We consider a UAV network which has information on user hotspot locations, and positions the UAVs accordingly. User locations can be determined using several methods, for example via self-reporting of GPS coordinates by the users themselves, or by measuring the user signal at several receivers with known locations (either ground infrastructure, or the UAVs themselves) and calculating the signal origin. If the number of hotspots is known a priori, then K-means clustering can be used to determine the hotspot centerpoints, as we have demonstrated in our prior work [16].

The UAV network positions the UAVs at a fixed height γ above ground, as depicted in Fig. 1. These UAVs may be positioned with respect to the user hotspot locations, or with respect to one another in a regular grid, depending on the UAV positioning strategy selected by the UAV network operator. We denote the set of UAVs as \( \Phi_u = \{x_0, x_1, \ldots\} \subset \mathbb{R}^2 \), where \( x_j \) corresponds to the projected coordinates of the \( j \)th UAV onto \( \mathbb{R}^2 \).

B. User Distribution

We consider a scenario where a number of users congregate in an area of interest, creating several user clusters which generate data demand. These clusters are referred to as user hotspots, which the UAV network attempts to serve. We model
the set of hotspots in the area of interest as a Matern Cluster Process (MCP) [26]. The number of user hotspots in the area of interest is random, with average hotspot density $\lambda_p$. The location of each hotspot is random in $\mathbb{R}^2$ and independent of the location of other hotspots; the set of hotspot centers is denoted as $\Phi_p = \{y_0, y_1, \ldots \} \subset \mathbb{R}^2$, where $y_i$ corresponds to the $i$th hotspot center. From the definition of the MCP, the set of hotspot centers $\Phi_p$ is a Poisson Point Process (PPP) with intensity $\lambda_p$. The users belonging to a hotspot $i$ are positioned in a circle of radius $r_{\text{max}}$ centered on the hotspot center $y_i$. Users are randomly and uniformly positioned inside this circle.

In an urban environment we expect that user hotspots will be restricted in size by buildings and other obstacles in the area: the hotspot radius $r_{\text{max}}$ of the MCP is intended to represent this effect and how it causes users to be concentrated in certain geographic areas [27]. We perform our analysis for a reference user, which is a randomly selected user of a randomly selected geographic area [27]. We denote $y_0 \in \Phi_p$ as the location of the reference user’s hotspot center. In this work we focus our attention on the downlink between a UAV and the reference user which it is serving, as the downlink is expected to be the performance bottleneck in a user hotspot [28].

C. Interference

The reference user will be served by one of the UAVs in $\Phi_u$. We assume full-buffer traffic with full frequency reuse, which results in all of the UAVs transmitting simultaneously on the same frequency bands. We also assume that the UAV network does not apply any interference mitigation techniques, and as a result each UAV other than the serving UAV can cause interference to the reference user.

In the scenario under consideration, the UAV network uses spectrum resources which are orthogonal to the spectrum used by terrestrial cellular networks. As a result of this, the UAV-user downlink is not affected by the underlying cellular network. This assumption is based on the on-going trend of allocating new spectrum bands exclusively for UAV communication [29], in addition to the allocation of new spectrum in the sub-6GHz bands for use by operators in next-generation cellular networks [30]. The UAV network under consideration takes advantage of these new spectral resources for communicating with the ground user, such that it does not experience interference from any terrestrial cellular infrastructure in the area.

D. Channel Propagation

As the UAV network operates in an urban environment, the wireless channel between a UAV and the reference user will be affected by several environmental factors. The distance between the transmitting UAV and the reference user will cause signal power attenuation. The position of a UAV relative to the reference user will determine the antenna gain of the received signal. The multipath behaviour of the signal will result in random fluctuations of the instantaneous signal strength. The buildings in the environment will block LOS between some of the UAVs and the reference user, which will create two distinct wireless channel types, LOS and NLOS, with their own signal attenuation and multipath fading behaviours [19], [22], [26].

1) LOS Blockage Probability Model: To model a distribution of buildings in the environment we adopt the model in [31], [22], which defines an urban environment as a collection of buildings arranged in a square grid. There are $\beta$ buildings per square kilometer, the fraction of area occupied by buildings to the total area is $\delta$, and each building has a height which is a Rayleigh-distributed random variable with scale parameter $\kappa$. Let $T_i \in \{1, n\}$ denote whether UAV $i$ has a LOS or NLOS channel type to the user. The probability of a UAV $i$ having LOS channel to the reference user ($T_i = 1$) is given in [31] as

$$P_1(r_i) = \prod_{n=0}^{\max(0,d-1)} \left(1 - \exp \left(-\frac{(\gamma - (n+1/2)\gamma)}{2\kappa^2} \right) \right)^n,$$

where $r_i = ||x_j||$ is the horizontal distance to the UAV and $d = \lfloor r_i \sqrt{3} \rfloor$. It follows that the NLOS probability $P_2(r_i) = 1 - P_1(r_i)$. A variety of LOS probability models have been considered by the wireless community, typically applied to scenarios where UAVs operate at heights in the order of several kilometers. Our choice of this model is motivated by our prior work [24], where we evaluated its suitability for describing low-altitude UAV small cell networks.

2) Transmit Power and Antenna Gain: We assume UAVs have identical transmit power $\mu$ and a directional antenna with beamwidth $\omega$. The main beam illuminates the area directly beneath the UAV. We assume a uniform and rotationally symmetric beam pattern; using the approximations (2-26) and (2-49) in [32] and assuming perfect antenna radiation efficiency the antenna gain $\eta$ in the direction of the reference user from UAV $i$ can be expressed as

$$\eta = \begin{cases} \mu 16\pi/(\omega^2), & \text{if } r_i \leq u(\omega, \gamma), \\ 0, & \text{if } r_i > u(\omega, \gamma), \end{cases} \quad (2)$$

where $u(\omega, \gamma) = \tan(\omega/2)\gamma$. 
3) Received Signal Strength: The instantaneous received signal strength from a UAV \( i \) at \( x_i \) at the reference user is:

\[
S_i = \eta H_T l(r_i, \gamma, \alpha_{T_i}),
\]

where \( H_T \) is the Nakagami-m random multipath fading component experienced by the signal from UAV \( i \), \( l(r_i, \gamma, \alpha_{T_i}) = (\gamma^2 + \gamma^2)^{-\alpha_{T_i}/2} \) is the pathloss function, and \( \alpha_{T_i} \) is the pathloss exponent. Note that the channel type \( T_i \) will determine the value of the pathloss exponent \( \alpha_{T} \) as well as the multipath fading \( H_T \). In this work we model multipath fading using the Nakagami-m model, as this model allows for convenient mathematical analysis, while being able to represent a variety of radio environments [19][33]. For the special case where \( m_{T_i} = 1 \) the Nakagami-m model becomes equivalent to the Rayleigh model, whereas the Rician-K model with parameter \( K \) can be closely approximated by selecting \( m_{T_i} \), such that \( m_{T_i} = (K + 1)^2/(2K + 1) \) [33]. This allows us to align our model with empirically-validated models in [34], [35].

E. Serving UAV Selection

The serving UAV is the one from which the reference user observes the highest received power. The index of the serving UAV is a random variable which we denote as

\[
V = \arg\max_{i \in \Phi_x} \{S_i\},
\]

where \( \tilde{S}_i \) denotes the long-term average\(^1 \) power received from the \( i \)-th UAV located at \( x_i \in \Phi_x \). It follows that, given \( V = v \), the serving UAV location is denoted as \( x_v \), its horizontal distance to the user is \( r_v \), and so on.

The Signal-to-Interference-and-Noise Ratio (SINR) for the reference user can be described as:

\[
\text{SINR} = S_v/(I + \sigma^2),
\]

where, given \( V = v \), \( S_v = \eta H_T l(r_v, \gamma, \alpha_{T_v}) \) is the signal from the serving UAV a distance \( r_v \) away with channel type \( T_v \). \( I \) denotes the aggregate signal power received from all UAVs in \( \Phi_u \) other than the serving UAV \( v \), and \( \sigma^2 \) denotes the noise power.

The reference user is said to be successfully served by the UAV network if it establishes a downlink channel with a SINR above some minimum threshold \( \theta \). We refer to the probability of the SINR exceeding this threshold as the coverage probability

\[
P_c(\theta) = P(\text{SINR} > \theta).
\]

Using the Shannon capacity bound [36][Eq. 9.62] the Spectral Efficiency (SE) of the UAV network can be expressed in terms of the SINR as

\[
\text{SE} = E[\log_2(1 + \text{SINR})].
\]

\(^1\)Since cell-level association acts on the order of seconds, we assume that any fast fading effects (like the multipath fading) will be averaged out.

IV. MATHEMATICAL ANALYSIS

In this work we consider several UAV placement strategies, as this makes our analysis applicable to a wide variety of UAV network use cases. For the special case when the UAVs are positioned exactly above each hotspot center we derive analytical expressions for the coverage probability provided by the network of UAVs acting as small cell access points.

In subsection IV-A we explicitly define how the UAVs are positioned with respect to the reference user on the ground. The probabilistic distribution of the horizontal distances between UAVs and users forms the foundation of our entire analysis, as these distances determine the probability of LOS blockage, and ultimately which UAV the user associates with.

The reference user associates with the UAV with the strongest signal, following (4). For example, if the user associates to the UAV above its hotspot center then this is a consequence of the other UAVs being too far away, or being obstructed by buildings. In subsection IV-B we derive the expressions for the probability that the user associates to a certain candidate UAV, given the location of that candidate UAV and its channel type. These association probability expressions will form a part of the coverage probability expression which we derive at the end of this section.

In subsection IV-C we approach the random aggregate interference power. In our system model we describe the wireless channels as being affected by random Nakagami-m fading, independently of one another. Prior work from the wireless community has shown that, given this type of random fading, the coverage probability of a reference user can be expressed as a function of higher-order derivatives of the Laplace transform of the aggregate interference power [19]. In subsection IV-C we derive expressions for the Laplace transform of the aggregate interference and demonstrate how the higher-order derivatives can be obtained.

In subsection IV-D we bring our prior derivations together and produce an expression for the coverage probability of the reference user, in terms of the association probability, the higher-order derivative of the aggregate interference Laplace transform, and the distance distributions to the serving UAV.

A. UAV Placement and Distance Distribution

We consider a scenario where the UAV network serves the user hotspots by positioning exactly one UAV above the center of each hotspot. As a result, both the density and coordinates of the UAVs match those of the hotspots exactly, with \( \Phi_u = \Phi_p \) and \( \lambda_u = \lambda_p \). It follows that the reference user at the origin will have a UAV above its associated hotspot center; we denote this UAV with the index 0 and its location as \( x_0 \in \Phi_u \). We partition the set \( \Phi_u \) into the sets \( \{x_0\} \) and \( \Phi_u^0 = \Phi_u \setminus \{x_0\} \), containing the reference user hotspot UAV and all the remaining UAVs, respectively. Note that, following Slivnyak’s theorem [27][Theorem 8.10], the set of UAVs \( \Phi_u^0 \) remains a PPP with intensity \( \lambda_u \).

The horizontal distance between the reference user and the UAV at its hotspot center is denoted as random variable \( R_0 \). When the users in a hotspot are distributed according to an
MCP, the probability density function (pdf) of $R_0$ is provided in [26], as

$$f_{R_0}(r) = \begin{cases} 2r/r_r^2, & 0 \leq r \leq r_r^\text{max}, \\ 0, & \text{otherwise}, \end{cases} \quad (8)$$

where $r_r^\text{max}$ is the radius of a user hotspot.

The user may be served by one of the UAVs in $\Phi_u^t$ if it provides the strongest signal. The UAVs in the set $\Phi_u^t$ have a mixture of LOS and NLOS channels to the reference user; the subsequent derivations require us to consider the behaviour of LOS and NLOS UAVs in $\Phi_u^t$ separately. From the definition of the PPP, the set $\Phi_u^t$ can be separated into multiple independent PPP sets using a thinning procedure [27][Theorem 2.36]. With thinning, each UAV in the set $\Phi_u^t$ is removed (or "thinned") from the set with a certain probability. We use this thinning procedure to separate $\Phi_u^t$ into two disjoint sets, one which contains all the LOS UAVs, and the other all the NLOS UAVs. These sets are denoted as $\Phi_1 = \{ x_i \in \Phi_u^t : T_i = 1 \}$ and $\Phi_n = \{ x_i \in \Phi_u^t : T_i = n \}$, respectively, with the thinning probability being given by the LOS function $P_1(r)$. Both sets are PPP with intensity functions $\lambda_1(x) = P_1(\|x\|)\lambda$ and $\lambda_n(x) = P_n(\|x\|)\lambda$, respectively.

The pdf of the distance to the closest UAV in $\Phi_1$ and $\Phi_n$ is derived in [27] as

$$f_{R_j}(r) = 2\pi \lambda_j(r)r \exp \left( -2\pi \int_0^r \lambda_j(r)dr \right), \quad j \in \{1, n\}, \quad (9)$$

where $R_j$ denotes the distance to the closest UAV in $\Phi_j$.

B. Association Probability

If the user associates with a UAV in $\Phi_j$ it will associate to the closest UAV in the set, as all of the remaining UAVs in the set will, by definition, provide a weaker signal to the user. The pdf of the serving UAV distance will follow one of the distance distributions given in (8) and (9), depending on which UAV type the user associates with. The user will associate to a UAV of a given type if there are no other UAVs that provide a stronger signal to it. This is referred to as the association probability.

Conditioned on the serving UAV being a distance $r$ away from the user with a channel of type $t$, the association probability is the probability that the serving UAV is either the user hotspot UAV at $x_0$ or one of the remaining UAVs at $\Phi_t$.

**Proposition 1.** The probability that the user's serving UAV is its hotspot center UAV at $x_0$, when the serving UAV is a distance $r$ away with channel type $t$, is expressed as

$$A_0(t, r) = \mathbb{P}(V = 0 | T = t, R_0 = r) = \prod_{c_j(t,r)} \exp \left( -2\pi \int_0^r \lambda_j(z)dz \right). \quad (10)$$

Proof: The hotspot center UAV at $x_0$ will have the strongest received signal at the reference user if the closest UAVs in $\Phi_1$ and $\Phi_n$ are not close enough to provide a stronger signal. The probability $A_0(t, r)$ is then given as

$$A_0(t, r) = \mathbb{P}\left( (r^2 + \gamma^2)^{-\alpha/2} > \max\left( (R_1^2 + \gamma^2)^{-\alpha/2}, (R_n^2 + \gamma^2)^{-\alpha_n/2} \right) \right)$$

$$= \prod_{c_j(t,r)} \exp \left( -2\pi \int_0^r \lambda_j(z)dz \right). \quad (11)$$

where $a$ comes from the fact that $R_1$ and $R_n$ are distributed independently of each other and $b$ comes from the definition of the void probability of a PPP [27]. Here, $c_j(t,r)$ denotes the lower bounds on minimum distances the UAVs can have to the user while still giving a weaker signal than the serving UAV, which we can express as follows

$$c_j(t,r) = \begin{cases} r, & \text{if } j = t, \\ \min(u(\omega, \gamma), \sqrt{r^2 + \gamma^2}/\omega - \gamma^2), & \text{if } j = 1, t = n, \\ \sqrt{\max(0, (r^2 + \gamma^2)\alpha_j/\alpha_n - \gamma^2)}, & \text{if } j = n, t = 1. \end{cases} \quad (12)$$

**Proposition 2.** The probability that the serving UAV belongs to the set $\Phi_t$ is

$$A_i(t, r) = \mathbb{P}(V = i | T = t, R_t = r)$$

$$= \prod_{c_j(t,r)} \exp \left( -2\pi \int_0^r \lambda_j(z)dz \right) \mathbb{B}(t, r), \quad i \neq 0, j \neq t, \quad (13)$$

where $\mathbb{B}(t, r)$ denotes the probability that the hotspot center UAV at $x_0$ is providing a weaker signal than the serving UAV from $\Phi_t$, and is expressed as

$$\mathbb{B}(t, r) = 1 - \sum_{k \in \{1, n\}} \int_0^{c_k(t,r)} \mathbb{P}_k(z) f_{R_0}(z)dz. \quad (14)$$

Proof: $A_i(t, r)$ is the probability that there are no UAVs in the set $\Phi_j \cup \{x_0\}$, with $j \neq t$, which are close enough to the user to provide a stronger signal than the UAV a distance $r$ away with channel type $t$.

$$A_i(t, r) = \mathbb{P}\left( R_j > c_j(t,r) \right) \mathbb{P}(\bar{S}_0 < \bar{S}_i)$$

$$= \prod_{c_j(t,r)} \exp \left( -2\pi \int_0^r \lambda_j(z)dz \right) \left( 1 - \sum_{k \in \{1, n\}} \int_0^{c_k(t,r)} \mathbb{P}_k(z) f_{R_0}(z)dz \right). \quad (15)$$
where \((a)\) comes from the probability of the hotspot center UAV providing a weaker signal, which is equivalent to the probability that the hotspot center UAV is neither LOS and closer than \(c_l(t, r)\), nor NLOS and closer than \(c_n(t, r)\).

C. Laplace Transform of Aggregate Interference and Noise

In this subsection we provide an expression for the \(k\)-th derivative of the Laplace transform of the aggregate interference and noise \(L_{(I+\sigma^2)}\); this will be used for derivations in the next subsection.

**Proposition 3.** The \(k\)-th derivative of the Laplace transform \(L_{(I+\sigma^2)}\) is obtained as

\[
\frac{d^k L_{(I+\sigma^2)}(s)}{ds^k} = \sum_{i_0, i_1, i_2, i_3} \frac{k!}{i_0! i_1! i_2! i_3!} \frac{d^{i_0} L_{I_1}(s)}{ds^{i_0}} \frac{d^{i_1} L_{I_2}(s)}{ds^{i_1}} \frac{d^{i_2} L_{I_3}(s)}{ds^{i_2}} \frac{d^{i_3} \exp(-sa^2)}{ds^{i_3}},
\]

(16)

where \(L_{I_1}, L_{I_2}\) and \(L_{I_3}\) are the Laplace transforms of the aggregate interference from \(\Phi_1, \Phi_2\), and \(x_0\), respectively. The sum is over all the combinations of non-negative integers \(i_0, i_1, i_2, i_3\) that add up to \(k\).

**Proof:** The aggregate interference power \(I\) is the sum of the interference power \(I_1, I_2, I_3\) and \(I_0\) from the UAVs in \(\Phi_1, \Phi_2\), and \(x_0\), respectively. Recall that, from the definition of the thinning procedure [27][Theorem 2.36], the sets \(\Phi_1\) and \(\Phi_2\) are independent PPPs. This means that the number and position of interferers of one type has no impact on the number and position of interferers of the other type. As a result, the sums of the received interference signal powers \(I_1, I_2, I_3\) are random variables that are independently distributed with respect to one another, as well as \(I_0\). This means that the Laplace transform of the aggregate interference and noise \(L_{(I+\sigma^2)}\) can be represented as the product of Laplace transforms \(L_{I_1}, L_{I_2}, L_{I_3}\), and \(L_{I_0}\), as well as \(\exp(-sa^2)\). The derivative of \(L_{(I+\sigma^2)}\) can be expressed in the form given in (16) following the general Leibniz rule.

**Remark 1:** If the user is served by the hotspot center UAV at \(x_0 (V = 0)\) then \(I_0\) will be 0 and \(L_{I_0}\) will be 1, as the UAV will not create interference for itself.

The Laplace transforms \(L_{I_1}\) and \(L_{I_2}\) have been previously derived by us in [24], and the Laplace transform \(L_{I_0}\) for the case when the hotspot center interference \(I_0\) is not zero is given below.

**Proposition 4.** The Laplace transform of the interference \(I_0\), given \(V = i, T_V = t, R_V = r\), is given as

\[
L_{I_0}(s) = \frac{1}{(r_{\max})^2} B(t, r) \sum_{j \in \{i, r\}} (C_j(s) + D_j),
\]

(17)

where

\[
C_j(s) = \sum_{q = [c_j(t, r) \sqrt{s}]}^{\min(r_{\max}, u(\omega, \gamma)) \sqrt{s}} \mathbb{P}_j(l) \left( (u^2 - l^2) + \sum_{k=1}^{m_j} \left( \frac{m_j}{k!} \right) (-1)^k \left( (u^2 + \gamma^2)^{\frac{k}{2}} \frac{\Gamma(k, \alpha_j) + 1}{\eta s} \right) \right)
\]

\[
L_j(s) = \left( \frac{m_j}{k!} \right) (-1)^k \left( (u^2 + \gamma^2)^{\frac{k}{2}} \frac{\Gamma(k, \alpha_j) + 1}{\eta s} \right)
\]

(18)

For the case when \(c_j(t, r) < \min(r_{\max}, u(\omega, \gamma))\), with \(\mathbb{P}_j(l)\) denoting the Gauss hypergeometric function, \(l = \max(c_j(t, r), q/\sqrt{s})\) and \(u = \min(r_{\max}, u(\omega, \gamma), (q + 1)/\sqrt{s})\). If \(c_j(t, r) \geq \max(r_{\max}, \omega, (\omega, \gamma))\) then \(C_j(s) = 0\).

**Proof:** The proof is given in Appendix A.

D. General model

In this subsection we present the main analytical result of our paper.

**Theorem 1.** The coverage probability of the reference user served by a UAV network that positions itself above user hotspots is given as (20).

**Proof:** The proof is given in Appendix B.

For comparison against a network of UAVs positioned independently of user hotspots according to a PPP at a fixed height, we present the following corollary.

**Corollary 1.** The coverage probability of the reference user when served by a UAV network that is positioned according to a PPP independently of hotspot locations is given as

\[
\mathbb{P}_c(\theta, \gamma, \lambda, \omega) = \sum_{t \in \{i, n\}} \int_0^{\min(r_{\max}, u(\omega, \gamma))} \mathcal{A}_t(r) \left( \sum_{k=0}^{m-1} (-1)^k \frac{m_j}{k!} \frac{d^k L_{(I+\sigma^2)}(s_r)}{ds^k} \right) f_{R_t}(r) \, dr.
\]

(21)

**Proof:** If the UAV network is distributed independently of the hotspot centers then the reference user does not have a UAV above its hotspot center, and can only be served by a UAV from the set \(\Phi_0\). The expression in (21) is obtained by setting \(r_{\max} \to \infty\), which has the effect of setting \(f_{R_t}(r) \to 0\) for the range \(0 \leq r \leq u(\omega, \gamma), B(t, r)\) and \(L_{I_0} \to 1\) and which reduces the expression given in (20) to the one in (21). Note that (21) corresponds to the result presented by us in [24].
Remark 2: For our system model we have assumed that all of the user hotspots have the same radius \( r_{\text{max}} \). For certain scenarios this assumption can be relaxed. If the reference user belongs to a hotspot whose radius \( R_{\text{max}} \) is a random variable with some arbitrary pdf \( f_{R_{\text{max}}}(r_{\text{max}}) \), then the coverage probability of that reference user is given as

\[
P_c(\theta, \gamma, \lambda_u, \omega) = \int_0^{\infty} P_c(\theta, \gamma, \lambda_u, \omega, r_{\text{max}}) f_{R_{\text{max}}}(r_{\text{max}}) dr_{\text{max}}.
\]  

(22)

The reference user’s coverage probability is only impacted by the radius of its own hotspot, as that affects the pdf of the distance between the reference user and the hotspot center UAV \( R_0 \). The radii of the neighbouring hotspots do not affect the location of the hotspot centers (and therefore the neighbouring UAV locations).

V. Numerical Results

In this section we evaluate the performance of the UAV small cell network under multiple UAV placement strategies, using the mathematical expressions derived in the previous section as well as simulations in the R software environment. We consider a simulation window of 16 km\(^2\), with a 1 km\(^2\) area of interest in the center. The larger simulation window relative to the area of interest ensures that we eliminate any boundary effects that may be experienced by a user near the edge of the area of interest. We simulate random distributions of user hotspots inside the window. The UAVs are positioned according to one of several positioning strategies, and we calculate the SINR for the users inside the area of interest. This is repeated across 10,000 Monte Carlo (MC) trials, and from these trials we calculate the coverage probability values.

Unless stated otherwise the parameters used for the numerical results are from Table I. The values of the Nakagami-m fading parameters \( m \) and the path loss exponents \( \alpha \) are chosen to fall inside the range of values reported by the field trials in [35][37]. Fig. 2 shows a typical simulation of users and the hotspot centers where we can position the UAVs.

A. Model Performance Evaluation

In this subsection we consider UAVs positioned directly above user hotspot centers; we generate the following results using our mathematical expressions, and validate the accuracy of the derivations via simulations. In Fig. 3 to Fig. 11, solid lines denote the coverage probability obtained via Theorem 1, and the markers denote results from MC trials.

In Fig. 3 we demonstrate the performance of the UAV network for different values of the user hotspot radius \( r_{\text{max}} \). We can see that when the radius is the lowest, and therefore the users are the most concentrated, the performance of the network is best. This is due to the reduced distance between a typical user and the UAV above the hotspot center, which allows the user to associate to the hotspot center UAV more often and receive a better signal from it. For each hotspot radius there is a single UAV height which maximises the coverage probability: below this height the serving UAV is more likely to have NLOS to the reference user, which decreases the received signal strength, and above this height the interfering UAVs are more likely to have LOS on the user and more interfering UAVs will cast their antenna beam over the user, increasing interference. We note that the UAV height which

\[
\begin{align*}
P_c(\theta, \gamma, \lambda_u, \omega, r_{\text{max}}) = & \sum_{t \in \{1, n\}} \left( u(\omega, \gamma) \int_0^{A_0(t, r)} \sum_{k=0}^{m_t - 1} (-1)^k \frac{s_k^k}{k!} \frac{d^k L_{(t+\sigma^2)}(s_r)}{ds_r^k} P_t(r) f_{R_0}(r) dr + \right. \\
& \left. \int_0^{A_0(t, r)} \sum_{k=0}^{m_t - 1} (-1)^k \frac{s_k^k}{k!} \frac{d^k L_{(t+\sigma^2)}(s_r)}{ds_r^k} f_{R_t}(r) dr \right) .
\end{align*}
\]

\( (20) \)

\[
\begin{array}{|c|c|c|}
\hline
\text{Parameter} & \text{Symbol} & \text{Value} \\
\hline
\text{Antenna beamwidth} & \omega & 150^\circ \\
\text{LOS pathloss exponent} & \alpha_0 & 2.1 \\
\text{NLOS pathloss exponent} & \alpha_1 & 4 \\
\text{LOS Nakagami-m fading term} & m_0 & 3 \\
\text{NLOS Nakagami-m fading term} & m_1 & 1 \\
\text{UAV Transmit power} & \mu & 0.1 \text{W} \\
\text{Noise power} & \sigma^2 & 10^{-9} \text{W} \\
\text{Number of buildings per square km.} & \beta & 300 / \text{km}^2 \\
\text{Fraction of total area occupied by buildings} & \delta & 0.5 \\
\text{Building height scale parameter} & \kappa & 20 \text{m} \\
\text{SINR threshold} & \theta & 0 \text{dB} \\
\hline
\end{array}
\]

Table I. Numerical Result Parameters

\( \text{Fig. 2. A simulation of user hotspots in the area of interest. Black dots denote users; red marks denote MCP hotspot centers.} \)
maximises the coverage probability increases as we increase the hotspot radius; this is due to the fact that the increasing distance between a typical user and its hotspot center UAV increases the probability of a LOS-blocking building being in the way, and therefore the UAV network must increase its height to compensate. The dashed line denotes the coverage probability for the case where $r_{\text{max}} \to \infty$, which is equivalent to the performance of a UAV network that positions UAVs independently of the locations of the user hotspots. In Fig. 4 we present the spectral efficiency of the UAV network for the same parameters. We can see that the spectral efficiency curves closely match the shape of those given in Fig. 3, including the approximate locations of the optimum UAV height for each corresponding hotspot radius.

It is worth noting that the range of optimum UAV heights for the smaller hotspot radii in Fig. 3 and Fig. 4 is in the order of 25-50 m above ground. This height may be too low for feasible UAV network operation in an urban environment, due to factors such as wireless backhaul availability [25] or safety regulations [5], [6]. A possible solution is to design UAV antennas with narrower beamwidths to allow the UAVs to operate at higher altitudes, as shown in Fig. 5. We can see that decreasing the antenna beamwidth will have the effect of increasing the height the UAV network would need to operate at to maximise the coverage probability. This result matches our previously reported result in [24] for the case of an independently distributed UAV network.

In Fig. 6 we consider the network performance when the density of the user hotspots (and therefore the UAV network) is increased. We can clearly see that for greater hotspot densities the coverage probability deteriorates, due to the greater number of UAV interferers, which is not offset by the greater number of candidate serving UAVs for the user. It is also worth noting that the optimum UAV height appears to change very little for the different densities; following the results of the previous plots we conclude that the optimum height of a UAV network is primarily determined by the radius of the user hotspots, rather than the number of hotspots in a given area. This result is very different from the independent UAV placement case. Fig. 7 presents the coverage probability for the case when the UAV network is placed independently of the hotspots; we
can see that varying the UAV density will vary the optimum height, but the maximum achievable coverage probability is approximately the same, irrespective of UAV density.

In Fig. 8 we consider the effect of changing the UAV transmit power on the coverage probability of the network. We report that increasing the transmit power will only improve the network performance at low UAV heights, when the UAV network experiences NLOS channels. At greater heights the network becomes interference-limited: as a result any changes to the transmit power will affect the received signal strength of both the serving UAV signal as well as the interference signals, thus giving no coverage probability improvement.

In Fig. 9 we verify Remark 2. We consider user hotspots which have random radii, distributed according to a truncated Gaussian distribution (with positive, non-zero radii values). Comparing the numerical results we see that the randomness of the user hotspot radii does not appear to affect the overall coverage probability, as the results align very closely with the case where the hotspots all have the same radius.

In Fig. 10 we consider the network performance as we increase the proportion of area covered by buildings $\delta$. We see that denser environments help the network perform better at greater UAV heights, due to the buildings blocking interference signals from UAVs further away. At very low heights the performance is the same for all cases, as the limiting factor for the coverage probability is the small area illuminated by the UAV antennas, rather than building blockage.

In Fig. 11 we show the impact of the $\kappa$ building parameter on the network performance. We observe that the maximum achievable coverage probability does not change with the changing building heights, provided the UAVs adjust their own heights accordingly.

### B. UAV Placement Comparison

In this subsection we are interested in comparing the numerical performance of the UAV network, for different deployment strategies of the UAVs. In addition to placing the UAVs at hotspot centers as before, we consider a network where UAVs are positioned around user locations via an optimisation algorithm, and a network where UAVs are deployed according to a rectangular grid. Our optimisation algorithm of choice
Fig. 11. Coverage probability given a hotspot density of 25/km², as a function of the building parameter $\kappa$ and UAV height above ground. The x-axis shows the UAV height as a multiple of $\kappa$.

Fig. 12. Coverage probability for different UAV placement strategies, given a hotspot radius $r_{\text{max}}$ of 100 m and a UAV density of 5/km².

Fig. 13. Coverage probability for different UAV placement strategies, given a hotspot radius $r_{\text{max}}$ of 100 m and a UAV density of 25/km².

is the K-means clustering algorithm, which partitions a given set of points in space into cells of approximately equal size and finds the centroid of the cells. In our previous work [16] we have demonstrated how this algorithm can be used to optimally position UAVs around known user locations. Figs. 12 and 13 show how the three UAVs placement scenarios compare to one another, given different densities of user hotspots (and UAVs). Hotspot center results are obtained using the analytical expressions, while the K-means optimisation and grid deployment results are obtained via simulation.

The first result we note is that there is a significant degree of similarity in the performance resulting from the K-means optimisation algorithm, and our mathematical model where UAVs are positioned above the user hotspot centers. The second interesting result concerns UAV performance when the UAVs are deployed according to a rectangular grid, independently of user locations. For the low UAV density scenario in Fig. 12 we can see that putting UAVs at hotspot centers (or otherwise optimising the UAV locations with respect to user locations) gives the best overall performance, due to the shorter distance between users and their serving UAVs. The behaviour changes when we increase the UAV density in Fig. 13. In this case, even though this pattern does not take into account the actual locations of the users, at higher UAV densities the network benefits from a UAV placement strategy which maximises the distances between UAVs and which therefore limits the UAV coverage overlap and interference, even if this placement strategy does not necessarily reduce the distances between users and their serving UAVs. The K-means algorithm is able to give the best performance as it both minimises the distance between the users and their serving UAVs while also spreading the UAVs out across cells of roughly equal size, which separates out interferers.

VI. Conclusion

In this paper we have used stochastic geometry to model a UAV network serving user hotspots in an urban environment, considering UAV network parameters such as antenna beamwidth and height above ground, as well as environmental parameters such as user hotspot radius and building density. We derived an expression for the coverage probability of the UAV network as a function of these parameters and then verified the derivation numerically, while showing the trade-offs in performance that occur under different network conditions. We then compared this network performance against the case where the UAVs are positioned according to K-means, or according to a rectangular grid. Our results showed that positioning the UAVs above user hotspots is warranted when the density of hotspots (and therefore the density of required UAVs) is sufficiently low. For larger UAV densities, the inter-cell interference can negate the performance benefits of positioning the UAVs above user hotspots. As a result, the network can benefit more from positioning the UAVs in a grid pattern and thus spreading them out as much as possible to reduce interference.

In this paper and in our prior works we have demonstrated that stochastic geometry is a robust tool for analysing physical-layer network performance. While the focus of this work was on characterising and investigating the impact of intelligent
UAV placement on the resulting downlink performance, the mathematical contributions presented in this work can form the basis for additional types of UAV network analysis. For example, the impact of interference mitigation and UAV-terrestrial network coexistence can be modelled and analysed by relaxing the assumptions in Subsection III-C and adjusting our mathematical derivations accordingly. This can allow the interested reader to model the impact of interference mitigation techniques such as fractional frequency reuse on the achievable performance, or to analyse the performance trade-offs experienced by UAV and terrestrial network users when both tiers of networks co-exist on the same spectrum band.

Although our analysis was applied to characterise the downlink channel in the UAV network, the uplink channel of the UAV network can be characterised by following a similar analytical approach. By considering a scenario where a reference UAV receives a desired signal from a user on the ground, alongside interference from other users, the uplink performance can be described analytically using a similar process to the one presented in this paper.

**Appendix A**

In this appendix we derive the expression $\mathcal{L}_{l_0}(s)$, given $V = i, R_V = r, T_V = t$. For ease of notation we omit these conditional expressions from the derivations below. The Laplace transform $\mathcal{L}_{l_0}(s)$ is derived as

$$
\mathcal{L}_{l_0}(s) = \mathbb{E} \left[ \exp \left( -sI_0 \right) \right] = \mathbb{E}_{R_0, T_0} \left[ \mathbb{E}_{H_{R_0}} \left[ \exp \left( - H_{R_0} \eta l(R_0, \gamma, \alpha_{R_0}) s \right) \right] \right]
$$

$$
= \mathbb{E}_{R_0, T_0} \left[ g(R_0, s, m_{R_0}, \alpha_{R_0}) \right]
$$

$$
= \sum_{j \in \{1, \ldots, n\}} \int_0^{r_{\text{max}}} g(z, s, m_j, \alpha_j) \mathbb{P}(R_0 = z, T_0 = j | \bar{S}_0 < \bar{S}_i) dz,
$$

where

$$
g(z, s, m_j, \alpha_j) = \left( \frac{m_j}{\eta_s z^2 + \gamma^2} \right)^{\alpha_j/2 + m_j},
$$

which comes from $H_{R_0}$ being gamma distributed, with $\mathbb{P}(R_0 = z, T_0 = j | \bar{S}_0 < \bar{S}_i)$ denoting the joint probability of the hotspot center UAV’s distance and channel type, given that the hotspot UAV is positioned such that its signal is weaker than the user’s serving UAV signal. This is derived as

$$
\mathbb{P}(R_0 = z, T_0 = j | \bar{S}_0 < \bar{S}_i) = \frac{\mathbb{P}(R_0 = z, T_0 = j, \bar{S}_0 < \bar{S}_i)}{\mathbb{P}(\bar{S}_0 < \bar{S}_i)}
$$

and

$$
\mathbb{P}(\bar{S}_0 < \bar{S}_i | R_0 = z, T_0 = j) \mathbb{P}(T_0 = j | R_0 = z) \mathbb{P}(R_0 = z)
$$

where $(a)$ follows from replacing $\mathbb{P}(S_0 < S_i)$ with $\mathcal{B}(t, r)$ which is the probability of the hotspot center UAV providing a weaker signal than the serving UAV given in (14), $(b)$ follows from $\mathbb{P}(R_0 = z) = \int \mathcal{B}(t, r) dz$, $\mathbb{P}(T_0 = j | R_0 = z) = \mathbb{P}_j(z)$ and $\mathbb{P}(S_0 < S_i | R_0 = z, T_0 = j) = 1(c_j(t, r) \leq z \leq r_{\text{max}})$ where $1(.)$ is the indicator function. Inserting this expression into (23) we can write the integral as

$$
\frac{2}{(r_{\text{max}}^2) B(t, r)} \sum_{j \in \{1, \ldots, n\}} \int_{c_j(t, r)}^{r_{\text{max}}} g(z, s, m_j, \alpha_j) \mathbb{P}_j(z) dz.
$$

Recall that $\eta = 0$ for values of $z > u(\omega, \gamma)$, which will reduce $g(z, s, m_j, \alpha_j) \mathbb{P}_j(z) dz$ to 0. If $r_{\text{max}} > u(\omega, \gamma)$ then the above integral is separated into two sub-integrals as

$$
\frac{2}{(r_{\text{max}}^2) B(t, r)} \sum_{j \in \{1, \ldots, n\}} \left( \int_{c_j(t, r)}^{u(\omega, \gamma)} g(z, s, m_j, \alpha_j) \mathbb{P}_j(z) dz + \int_{u(\omega, \gamma)}^{r_{\text{max}}} g(z, s, m_j, \alpha_j) \mathbb{P}_j(z) dz \right).
$$

The integrals in (26) and (27) assume that $c_j(t, r) < \min(r_{\text{max}}, u(\omega, \gamma))$. For the case where $u(\omega, \gamma) \leq c_j(t, r) \leq r_{\text{max}}$ the first sub-integral in (27) reduces to 0 and the second sub-integral takes $c_j(t, r)$ as its lower integration bound, if $c_j(t, r) \geq \max(u(\omega, \gamma), r_{\text{max}})$ then both integrals reduce to 0. From the definition (1) the LOS probability is a step function, therefore the integral

$$
\int_{c_j(t, r)}^{r_{\text{max}}} g(z, s, m_j, \alpha_j) \mathbb{P}_j(z) dz
$$

can be written as a sum of weighted integrals

$$
\sum_{q = [c_j(t, r) \sqrt{\gamma}]}^{u(\omega, \gamma)} \mathbb{P}_j(l) \int_{c_j(t, r)}^{l} g(z, s, m_j, \alpha_j) dz,
$$

where $l = \max(c_j(t, r), q/\sqrt{\gamma})$ and $u = \min(u(\omega, \gamma), (q + 1)/\sqrt{\gamma})$. Using a derivation process similar to (11) in [24] the integral $\int_{c_j(t, r)}^{u} g(z, s, m_j, \alpha_j) dz$ can be expressed in analytical form.

$$
\int_{c_j(t, r)}^{u} \left( \frac{m_j}{\eta_s (z^2 + \gamma^2)^{\alpha_j/2 + m_j}} \right)^{m_j} dz = \frac{a}{(a^2 + \gamma^2)^{1/2}} \int_{1/(a^2 + \gamma^2)^{1/2}}^{1} \left( \frac{m_j}{\eta_s (z^2 + \gamma^2)^{\alpha_j/2 + m_j}} \right)^{m_j} dz
$$

and

$$
\int_{c_j(t, r)}^{u} \left( \frac{1}{1 + \omega m_j (\eta_s)^{-1}} \right)^{m_j} w^{2/\alpha_j - 1} dw.
$$
\[ \frac{1}{\alpha_j} \left( \int_{(u^2 + \gamma^2)^{a_j/2}}^{(u^2 + \gamma^2)^{a_j/2}} u^{2/\alpha_j - 1} du \right) \]
\[ + \sum_{k=1}^{m_j} \left( \frac{m_j}{k} \right) (-1)^k \int_{(u^2 + \gamma^2)^{a_j/2}}^{(u^2 + \gamma^2)^{a_j/2}} u^{2/\alpha_j - 1} \left( 1 + \frac{\gamma}{m_j (\gamma^2 a_j/2)} \right)^k du \]
\[ = \frac{1}{2} \left( u^2 - l^2 \right)^2 \frac{1}{\alpha_j} \left( \frac{m_j}{k} \right) (-1)^k \left( \frac{(u^2 + \gamma^2)^{a_j/2}}{\gamma} \right) \]
\[ - \left( l^2 + \gamma^2 \right)^2 \frac{1}{\alpha_j} \left( \frac{m_j}{k} \right) (-1)^k \left( \frac{(l^2 + \gamma^2)^{a_j/2}}{\gamma} \right) \] (29)

where (a) stems from the substitution \( u = (z^2 + \gamma^2)^{1/2} \), (b) from the substitution \( w = y^2 \), (c) applying binomial expansion and (d) from using [38][Eq. 3.194.1]. The solution above is inserted into (28) which is denoted as \( C_j(s) \) and inserted into (17).

### Appendix B

The coverage probability expression (20) is derived as follows:

\[ \frac{P[\eta/(R_V, \gamma, \alpha_{TV}) \geq \theta]}{I + \sigma^2} = P[H_{TV} \geq \frac{\theta(I + \sigma^2)}{\eta(R_V, \gamma, \alpha_{TV})}] \]
\[ = E \left[ \frac{\Gamma(m_{TV}, s_{RV}(I + \sigma^2))}{\Gamma(m_{TV})} \right] \]
\[ = E \left[ \exp(-s_{RV}(I + \sigma^2)) \sum_{k=0}^{m_{TV}-1} \frac{(s_{RV}(I + \sigma^2))^k}{k!} \right] \]
\[ = E \left[ \sum_{k=0}^{m_{TV}-1} (-1)^k \frac{s_{RV}^k}{k!} \frac{d^k}{ds_{RV}^k} \frac{1}{\Gamma(m_{TV})} \right] \]
\[ = E \left[ \frac{m_{TV}!}{s_{RV}^{m_{TV}}(I + \sigma^2)^{m_{TV}}(I + \sigma^2)^k} \right] \] (30)

where (a) comes from the random fading \( H_{TV} \), being gamma distributed with channel-dependent fading parameter \( m_{TV} \) with \( \Gamma(\cdot) \) and \( \Gamma(\cdot, \cdot) \) being the gamma and upper incomplete gamma functions, respectively, where \( s_{RV} = m_{TV} \theta/(\eta(R_V, \gamma, \alpha_{TV})) \), (b) comes from expressing the upper incomplete gamma function as in \([38][8.352.2] \), (c) arises from the substitution \( \exp(-s_{RV}(I + \sigma^2))((I + \sigma^2))^{k} = (-1)^k \frac{d^k}{ds_{RV}^k} \exp(-s_{RV}(I + \sigma^2)) / \Gamma(m_{TV}) \), (d) comes from the Leibniz integral rule.

Let

\[ g(T_V, R_V) = \sum_{k=0}^{m_{TV}-1} (-1)^k \frac{s_{RV}^k}{k!} \frac{d^k}{ds_{RV}^k} \frac{1}{\Gamma(m_{TV})} \] (31)

We can express (30) in the final form given in (20) using the following procedure

\[ = E \left[ g(T_V, R_V) \right] \]
\[ = E \left[ \mathbb{E}_V [g(T_V, R_V)] \right] \]
\[ = P[V = 0 | T_0, T_1, R_0, R_1] g(T_0, R_0) \]
\[ + P[V = i | T_0, T_1, R_0, R_1] g(T_i, R_i) \]
\[ = E_{T_0, R_0} \left[ E_{T_1, R_1} \mathbb{P}[V = 0 | T_0, T_1, R_0, R_1] g(T_0, R_0) \right] \]
\[ + E_{T_1, R_1} \left[ E_{T_0, R_0} \mathbb{P}[V = i | T_0, T_1, R_0, R_1] g(T_0, R_0) \right] \]
\[ = E_{T_0, R_0} \left[ P[V = 0 | T_0, R_0] g(T_0, R_0) \right] \]
\[ + E_{T_1, R_1} \left[ P[V = i | T_1, R_1] g(T_1, R_1) \right] \]
\[ = \mathbb{E}(\omega, \gamma) \int_{0}^{1} (\mathcal{A}_0(1, r) g(1, r)) \mathcal{P}_2(\cdot) f_{R_0}(r) dr \]
\[ + \mathbb{E}(\omega, \gamma) \int_{0}^{1} (\mathcal{A}_1(1, r) g(1, r)) \mathcal{P}_2(\cdot) f_{R_1}(r) dr \] (32)

where in (a) \( T_0 \) and \( R_0 \) denote the channel type and distance of the hotspot center UAV while \( T_1 \) and \( R_1 \) refer to the channel type and corresponding distance of another candidate serving UAV, (b) comes from \( \mathbb{P}[V = u | T_0 = 1, R_0 = r] = \mathcal{A}_1(t, r) \) defined in Subsection IV-B and the expectation over \( R \) and \( T \) are found by applying the pdfs defined in Subsection IV-A. By rearranging the order of summation and integration we arrive at the final form given in (20). Note that, following the antenna gain definition in (2), a UAV will have a gain of 0 at the reference user if it is further away than \( u(\omega, \gamma) \); as such we consider \( u(\omega, \gamma) \) to be the upper limit on the serving UAV distance in the integral above.

### References


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