Multi-carrier Modulation and OFDM

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Multi-carrier systems: basic idea

- Typical mobile radio channel is a fading channel that is flat or frequency selective
- For high bandwidth applications channel is frequency selective and delay spread dictates throughput
- Multicarrier modulation is a technique where multiple low data rate carriers are combined by a transmitter to form a composite high data rate transmission
- In a classic multi-carrier system, the available spectrum is split into several non-overlapping frequency sub channels. The individual data elements are modulated into these sub channels and are thus frequency multiplexed
Multi-carrier transmission

- Converts a high-data rate bit stream into multiple lower-data rate substreams
- Each substream is modulated onto a different carrier

Advantage

- Since symbols are transmitted at a lower rate, the effects of delay spread are reduced
  - Reduced inter-symbol interference
- This in turn reduces the complexity of the equalizer
Robustness to delay spread

- MC-modulation increases the symbol time by modulating into narrow sub-channels

Channel frequency responses for a single carrier and multicarrier system. In the multicarrier system each sub channel only undergoes slight distortion

Orthogonal Frequency Division Multiplexing

- In classic multicarrier system guard bands have to be inserted, resulting in poor spectral efficiency
- A more efficient approach is to allow the spectra of individual subcarriers to overlap
- Problem: If individual subcarriers are overlapping isn’t there interference between carriers?
- Answer: No! If subcarrier tones are separated by the inverse of the signaling symbol duration, independent separation of frequency multiplexed tones is possible
  - This ensures that the spectra of individual sub channels are zeros at other subcarrier frequencies
OFDM carrier

- Orthogonal waveforms are generated by using signals that have integer number of cycles in the duration $T_s$

![Subcarriers in OFDM]

$T_s$

OFDM symbols

- Consider $N_c$ complex-valued source symbols: $S_n, n = 0, 1, ..., N_c - 1$
- These symbols are transmitted in parallel using $N_c$ sub-carriers
  - All of these symbols combined are referred to as an OFDM symbol
- If the source symbol duration is $T_d$, then the OFDM symbol duration is $T_s = N_cT_d$
- The $N_c$ sub-carriers have a spacing of $f = \frac{1}{T_s}$
OFDM carriers

• The baseband information of the $k^{th}$ carrier can be expressed as

$$ (x_k + jy_k)(\cos 2\pi kf + j\sin 2\pi kf) $$

• The OFDM signal is the sum of all the signals in each of its subcarriers which can be written as (usually implemented using IFFT)

$$ s(t) = \sum_{k=0}^{N_t-1} (x_k + jy_k)(\cos 2\pi kf + j\sin 2\pi kf) $$

Recovering the individual symbols

• The individual modulated symbols at the receiver are recovered using the FFT

• The $k^{th}$ output from the FFT is:

$$ z_k = \int_0^T s(t)(\cos 2\pi kf - j \sin 2\pi kf) \, dt = $$

$$ \sum_{n=0}^{N_t-1} \int_0^T (x_n + jy_n) \cos 2\pi mf (\cos 2\pi kf - j \sin 2\pi kf) \, dt + \int_0^T (x_n + y_n) \sin 2\pi mf (\cos 2\pi kf - j \sin 2\pi kf) \, dt $$
Solving these integrals...

- Trigonometry reminder:

\[
2 \cos A \cos B = \cos(A - B) + \cos(A + B)
\]

\[
2 \sin A \sin B = \cos(A - B) - \cos(A + B)
\]

\[
2 \sin A \cos B = \sin(A - B) + \sin(A + B)
\]

OFDM spectrum

- The individual spectra of the subcarriers are sinc functions
- Zero crossings occur at every integer multiple of \( f \) and hence no Inter-Carrier Interference occurs in the frequency domain
- Note the analogy with time-domain sinc pulses
Spectral efficiency

- For $N$ sub-carriers, the bandwidth of conventional FDM is $2N/T$ while that of OFDM is $(N+1)/T$. By allowing the sub-carrier spectra to overlap, OFDM improves the spectral efficiency.

Power spectral density example ($N_c = 16$)
Guard interval

• As $N_c$ increases, the OFDM symbol duration $T_s$ becomes large as compared to the duration of the impulse response $\tau_{\text{max}}$ of the channel.
• To completely eliminate ISI, must add a guard interval $T_g \geq \tau_{\text{max}}$.
• The new duration of the OFDM symbol is then

$$T'_s = T_s + T_g$$

Sampled sequence with cyclic guard extension

• The length of the guard interval $L_g$ must be

$$L_g \geq \left\lceil \frac{\tau_{\text{max}} N_c}{T_s} \right\rceil$$

• The sampled signal with the guard extension becomes

$$x_v = \frac{1}{N_c} \sum_{n=0}^{N_c-1} S_n e^{j 2 \pi n v / N_c}, \quad v = -L_g, \ldots, N_c - 1$$
OFDM transmitter and receiver

Matrix notation

- Complex-valued source symbols, transmitted in parallel as an OFDM symbol
  \[ s = \begin{pmatrix} S_0 & S_1 & \cdots & S_{N_c-1} \end{pmatrix} \]

- \( N_c \times N_c \) channel matrix
  \[ H = \begin{pmatrix} H_{0,0} & 0 & \cdots & 0 \\ 0 & H_{1,1} & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{N_c-1,N_c-1} \end{pmatrix} \]

(Why is this a diagonal matrix?)
Matrix notation (cont’d)

- Additive noise
  \[ n = \begin{pmatrix} N_0 & N_1 & \ldots & N_{N_c-1} \end{pmatrix}^T \]

- Received signals
  \[ r = \begin{pmatrix} R_0 & R_1 & \ldots & R_{N_c-1} \end{pmatrix}^T \]

\[ r = Hs + n \]

An OFDM frame

(a) OFDM symbol

(b) OFDM frame
OFDM advantages

• High spectral efficiency for large number of subcarriers: nearly rectangular frequency-domain representation of signal
• Low-complexity receivers: due to low ICI and ISI if guard interval is long enough
• Flexible spectrum adaptation: good for DSA
• Different modulation can be applied to different subcarriers to suit the transmission conditions on each subcarrier

OFDM disadvantages

• High peak-to-average power ratio (PAPR): requires highly linear power amplifiers
• Some loss of spectral efficiency due to guard interval
• Average frequency and time synchronization is required
• More sensitive to Doppler spread than single-carrier systems
**OFDM example: DVB-T**

<table>
<thead>
<tr>
<th></th>
<th>8 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bandwidth</strong></td>
<td></td>
</tr>
<tr>
<td><strong># of carriers</strong></td>
<td>1705 (2k FFT)</td>
</tr>
<tr>
<td></td>
<td>6817 (8k FFT)</td>
</tr>
<tr>
<td><strong>Symbol duration</strong> $T_s$</td>
<td>224 $\mu$s</td>
</tr>
<tr>
<td></td>
<td>896 $\mu$s</td>
</tr>
<tr>
<td><strong>Carrier spacing</strong> $F_s$</td>
<td>4.464 kHz</td>
</tr>
<tr>
<td></td>
<td>1.116 kHz</td>
</tr>
<tr>
<td><strong>Guard time</strong> $T_g$</td>
<td>$T_s/32, T_s/16, T_s/8, T_s/4$</td>
</tr>
<tr>
<td><strong>Modulation</strong></td>
<td>QPSK, 16-QAM, 64-QAM</td>
</tr>
<tr>
<td><strong>FEC coding</strong></td>
<td>Reed Solomon + convolutional with code rate ½ up to 7/8</td>
</tr>
<tr>
<td><strong>Max. data rate</strong></td>
<td>31.7 Mbps</td>
</tr>
</tbody>
</table>

**OFDM example: IEEE 802.11a**

<table>
<thead>
<tr>
<th></th>
<th>20 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bandwidth</strong></td>
<td></td>
</tr>
<tr>
<td><strong># of carriers</strong></td>
<td>52 (64 FFT)</td>
</tr>
<tr>
<td><strong>Symbol duration</strong> $T_s$</td>
<td>4 $\mu$s</td>
</tr>
<tr>
<td><strong>Carrier spacing</strong> $F_s$</td>
<td>312.5 kHz</td>
</tr>
<tr>
<td><strong>Guard time</strong> $T_g$</td>
<td>0.8 $\mu$s</td>
</tr>
<tr>
<td><strong>Modulation</strong></td>
<td>BPSK, QPSK, 16-QAM, 64-QAM</td>
</tr>
<tr>
<td><strong>FEC coding</strong></td>
<td>Convolutional code with rate ½ up to 3/4</td>
</tr>
<tr>
<td><strong>Max. data rate</strong></td>
<td>54 Mbps</td>
</tr>
</tbody>
</table>
**OFDM example: IEEE 802.16a**

<table>
<thead>
<tr>
<th>Bandwidth</th>
<th>From 1.5 to 28 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td># of carriers</td>
<td>256</td>
</tr>
<tr>
<td>Symbol duration $T_s$</td>
<td>from 8 to 125 $\mu$s (depending on the bandwidth)</td>
</tr>
<tr>
<td>Guard time $T_g$</td>
<td>from $1/32$ up to $\frac{1}{4}$ of $T_s$</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK, 16-QAM, 64-QAM</td>
</tr>
<tr>
<td>FEC coding</td>
<td>Reed Solomon + convolutional code with rate $\frac{1}{2}$ up to $\frac{5}{6}$</td>
</tr>
</tbody>
</table>

**OFDM applications**

- **Wireline**
  - Asymmetric Digital Subscriber Loop (ADSL)
- **Wireless**
  - Digital Audio Broadcasting (DAB)
  - Digital Video Broadcasting-Terrestrial (DVB-T)
  - Integrated Services Digital Broadcasting-Terrestrial (ISDB-T)
  - Wireless LAN (IEEE 802.11(a), HiperLAN/2)
  - Wireless MAN (IEEE 802.16 a/b)