Modulation (6):
Constellation Diagrams

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Geometric representation of modulation signal

- Digital Modulation involves
  - Choosing a particular signal waveform for transmission of a particular symbol
  - For M possible symbols, we thus have M possible waveforms

\[ S = \{s_1(t), s_2(t), \ldots, s_M(t)\} \]

- For binary modulation, each bit is mapped to a signal from a set of signals S, that has two signals
- We can view the elements of S as points in a vector space
Vector space

- We can represent the elements of S as linear combination of basis signals.

- The number of basis signals is the dimension of the vector space.

- Basis signals are orthogonal to each other.

- Each basis is normalized to have unit energy:

\[
E = \int_{-\infty}^{\infty} \phi_i^2(t) dt = 1
\]

\(\phi_i(t)\) is the \(i^{th}\) basis signal.
Example: BPSK

\[ s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b \]

\[ s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b \]

\[ \phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \]

\[ S = \left\{ \sqrt{E_b} \phi_1(t), -\sqrt{E_b} \phi_1(t) \right\} \]

Two signal waveforms to be used for transmission

The basis signal

Constellation Diagram (Dimension = 1)
Constellation diagram

- Properties of modulation scheme can be inferred from *constellation diagram*

- *Bandwidth* occupied by the modulation decreases as the number of signal points per dimension increases (getting *more dense*)

- *Probability of bit error* is inversely proportional to the distance between the closest points in the constellation
  - Bit error decreases as the distance increases (*sparse*)
Demodulation

- In ideal coherent detection, prototypes of the possible arriving signals are available at the receiver.
  - These *prototype waveforms* exactly replicate the signal set.
  - The receiver is then said to be *phase-locked* to the transmitter.

- During detection, the receiver multiplies and integrates (*correlates*) the incoming signal with each of its prototype replicas, and then decides for the most correlated replica among those available.
Amplitude and phase

- Amplitude and phase can be modulated simultaneously and separately, but this is difficult to generate, and especially difficult to detect.

- Instead, in practical systems the signal is separated into another set of independent components: $I$ (Inphase) and $Q$ (Quadrature).

- These components are orthogonal and do not interfere with each other.

- For representation of these components, we use phasors.
Phasor representation

- A sine wave can also be represented as a phasor (equivalent to a vector in polar form)
Phasor representation (cont’d)

- A phasor is a rotating vector

\[ \text{time } t = t_2 \]
Back to modulation

- We can think of modulation using phasor/polar format, rather than...
“Speed” with which one walks the whole circle.
In-phase and quadrature

\[ ACos(2\pi f_c t + \phi) = ACos(2\pi f_c t)\cos(\phi) - ASin(2\pi f_c t)\sin(\phi) \]

\[ I = ACos(\phi) \]
\[ Q = ASin(\phi) \]

\[ ACos(2\pi f_c t + \phi) = ICos(2\pi f_c t) - QSin(2\pi f_c t) \]

I = Amplitude of the “In-Phase Carrier”
Q = Amplitude of the “Quadrature Phase Carrier”
In-phase and quadrature (cont’d)
**Constellation points**

- Most digital modulations map the data to a number of discrete points on the $I/Q$ plane.

- These are known as *constellation points*.
PSK

BPSK
One Bit Per Symbol

QPSK
Two Bits Per Symbol
QAM

Constellation Diagram

32QAM
Five Bits Per Symbol
Symbol Rate = 1/5 Bit Rate
I/Q modulator

- I/Q diagrams are particularly useful because they mirror the way most digital communications signals are created using an I/Q modulator.

- In the transmitter, I and Q signals are mixed with the same local oscillator (LO).

- A 90 degree phase shifter is placed in one of the LO paths.
  - Signals that are separated by 90 degrees are also known as being orthogonal to each other or in quadrature.
I/Q modulator block diagram
**I/Q demodulator**

- The composite signal with magnitude and phase (or $I$ and $Q$) information arrives at the receiver input.

- The input signal is mixed with the local oscillator signal at the carrier frequency in two forms:
  - One is at an arbitrary zero phase.
  - The other has a 90 degree phase shift.
I/Q demodulator (cont’d)

- Signals that are in quadrature do not interfere with each other
  - They are two independent components of the signal

- The main advantage of I/Q modulation is the ease of combining independent signal components into a single composite signal and later splitting such a composite signal into its independent component parts
  - Exploitation of the symmetry due to the trigonometric identities