Marginal Power Efficiency Considerations in Power Control Adaptations for Multihop Cellular Networks

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***Abstract*—We propose a Marginal Power Efficiency-based Adaptation (MPEA) scheme for distributed power control in wireless communication systems. Nodes autonomously adapt their transmit power keeping a differential measure of power efficiency, defined as the additional capacity obtained per unit of transmit power, at a desirable threshold. We combine the MPEA with the Foschini-Miljanic (FM) algorithm [1] into a power control algorithm for a multihop cellular network where nodes can dynamically adapt to both local and global conditions. We derive a fixed point to which the power allocations converge under the scheme and illustrate a significant improvement in the system throughput.**

***Index Terms*—Heterogeneous Traffic, Opportunistic Power Control, End-to-End Goals**

I. Introduction

LTE-Advanced networks support the use of relays to improve system capacity and coverage [2]. When coupled with power control, this motivates the need for autonomous, distributed and flexible link adaptation schemes [3], which can support heterogeneous traffic and balance spectral efficiency and energy consumption [4].

In this paper, we first propose a distributed power control where each radio adapts its transmit power to control the marginal capacity improvement with respect to its power, referred to as the *marginal power efficiency*. This metric is the differential counterpart to *power efficiency*, which is defined as capacity per unit power [5]. It is motivated by the inherent tradeoff between the transmit power (cost) and the data rate (utility) of a link with a given bandwidth and effective interference. Our proposed power control mechanism falls under the category of *opportunistic power control* (OPC), used in [6–9]. OPC is suitable for data services where maintaining a constant SINR is not necessary and variation in latency can be tolerated. We show that our proposed power control scheme significantly increases network throughput with lower average transmit power than both the OPC algorithm in [6][7] and the well-known Foschini-Miljanic (FM) algorithm [1].

We design an adaptive power control scheme for a two-hop cellular network, with at most one relay between the user equipment (UE) and the Base Station.

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We call the proposed method Multihop Marginal Power Efficiency-based Adaptation (MH-MPEA). Under our power adaptation scheme, the links to the Base Station maintain a target marginal power efficiency. However, if a relay-to-base station link is congested, some UE-to-relay links instead minimize their transmit power (and reduce interference in the network), subject to a minimum SINR, using the FM power control. Thus, the power allocation objective of autonomous UEs would vary with the current network-wide conditions, unlike in [10]. We also derive a closed-form expression for the converged transmit power levels of the radios.

We show that, when all links adapt under the scheme, the system throughput is increased by at least 200%, at a cost of less than 15% *infeasible* links [11, 12], over a range of simulation scenarios. This is even though UEs adapt to optimize their own local objective instead of some network-wide goal as in [8, 9, 13, 14].

The paper is organized as follows. In Section II, we present the system model, which is followed by the problem formulation in Section III. In Section IV, we present the network adaptation scheme. In Section V we derive convergence conditions, followed by Section VI, where we discuss system feasibility and fairness. In Section VII, we then present the simulation results. Finally, Section VIII summarizes the contributions of the paper.

II. System Model

Consider a single-cell network serving n user equipment (UEs) that wish to communicate with the Base Station (BS) on the uplink. There are N_r fixed relay stations (RS), which are located towards the edges of the cell to provide coverage for UEs farther away from the base station (see Fig. 1 with $N_r = 3$). Each RS receives data from those UEs which fall in its coverage region and decodes-and-forwards the data to the BS. Those UEs which uplink to a relay use a multihop route to connect with the BS. All established uplinks are power- and rate-adaptive and can be represented by a set $\mathcal{L} = \{1, 2, \cdots, L\}$ which is the union of three subsets $\mathcal{L}_1$ (UE-to-RS), $\mathcal{L}_2$ (RS-to-BS) and $\mathcal{L}_3$ (UE-to-BS).

We assume that there are multiple channels that can be conceptualized over time or frequency, similar to *resource blocks* in LTE [2]. Each link transmits over a single channel of bandwidth W Hz. Channel assignments are made with the constraint that all links to and from a given Point of Access (relay or BS) operate on orthogonal channels. With the same constraint for every node, these channels can be *reused* by other links in the cell (e.g. as in Fig. 1). We represent the channel assignment as a set $\mathcal{F}(f) \subseteq \mathcal{L}$ for each of the F channels $f \in \{1, 2, \cdots, F\}$.

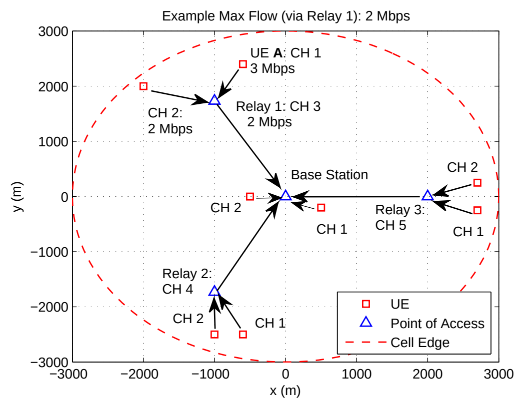

Fig. 1: A depiction of a multihop cellular network with assigned channels. Local adaptations take into account the end-to-end data rate performance.

Let $g_{j,i}$ represent the channel gain between the transmitter of link j and the receiver of link i, which depends on factors such as the path loss and fading. A cross-link gain matrix $\mathbf{F}$ of dimension $L \times L$ is defined as

$$\mathbf{F}(i,j) = \begin{cases} 0 \text{ if } i = j \text{ or } i,j \text{ on orthogonal channels,} \\ \frac{g_{j,i}}{g_{i,i}} \text{ otherwise.} \end{cases} \tag{1}$$

The transmit powers of all links are represented by the vector $\mathbf{P} = [P_1, P_2, \cdots, P_L]$. The effective interference E_i for a link between i and j that operates on channel f is defined as [6]

$$E_i = \frac{n_o + \sum_{j \neq i, j \in \mathcal{F}(f)} g_{j,i} P_j}{g_{i,i}}, \tag{2}$$

where n_o is the total thermal noise power per channel. Let $\mathbf{P}_{\max}$ and $\mathbf{D}$ be $L \times 1$ vectors, where the first vector represents the links' maximum transmit powers and the i^{th} element of the second vector is $\mathbf{D}(i) = n_o / g_{i,i}$. The achieved SINR of link i is $\gamma_i = \frac{P_i}{E_i}$. The corresponding data rate of link i is assumed to be

$$\eta_i = W . \log_2(1 + \gamma_i) \text{ bits/s.} \tag{3}$$

III. Problem Formulation

Consider the aggregate end-to-end data rate, denoted as η_N. Using the Max-Flow Min-Cut Theorem [15], this can be expressed as:

$$\eta_N = \sum_{i \in \mathcal{L}_3} \eta_i + \sum_{\forall r \in \mathcal{L}_2} \min\left(\eta_r, \sum_{j \in \mathcal{L}_1^r} \eta_j\right) \tag{4}$$

where $\mathcal{L}_1^r \subseteq \mathcal{L}_1$ denotes the subset of UE-to-RS links forwarding their data via the backhaul RS-to-BS link $r \in \mathcal{L}_2$. The minimum of the outgoing RS-to-BS link data rate η_r ($r \in \mathcal{L}_2$) and the aggregate incoming data rate $\sum_{j \in \mathcal{L}_1^r} \eta_j$ is the maximum aggregate data rate possible for the flows (see Fig. 1 as an example). When $\eta_r < \sum_{j \in \mathcal{L}_1^r} \eta_j$, the backhaul RS-to-BS link becomes the *bottleneck* link for all UEs of relay r, as it constrains their aggregate end-to-end data rate. We define the *rate differential* for each relay r as

$$V_r = \eta_r - \sum_{j \in \mathcal{L}_1^r} \eta_j. \tag{5}$$

We propose that nodes that have direct links to the base station maximize their data rate. Conversely, if a node is communicating with the BS via a relay, then its power control objective should also depend on the backhaul capacity. If the RS-to-BS link is congested (as indicated by V_r), an associated UE-to-RS link could operate in power minimization mode, subject to some SINR constraint, to alleviate the backhaul link. Therefore, we define the objective function of the transmitter of each link i as

$$\begin{aligned} \underbrace{\max}_{P_i} \; & w_i \eta_i + (1 - w_i)(P_{max,i} - P_i) \\ \text{s.t. } & (1 - w_i)\gamma_i \geq (1 - w_i)\beta \\ & w_i \frac{\partial \eta_i}{\partial P_i} \geq w_i \mu_i \\ & P_i \leq P_{\max,i}, \forall i \\ & w_i \in \{0, 1\}, \forall i \end{aligned} \tag{6}$$

If $w_i = 1$, then link i maximizes its data rate subject to a minimum marginal capacity improvement with respect to its power μ_i, referred to as the *marginal power efficiency*. In contrast, if $w_i = 0$, then link i minimizes its transmit power subject to some minimum SINR threshold β (i.e. feasibility constraint). The power allocations are subject to a maximum transmit power constraint. We illustrate how $w_i \in \{0, 1\}$ is determined for each link in the next section.

Algorithm 1: The determination of w_i for each link i is based on the network-wide conditions.

```
Initialization: w_i(0) = 1, ∀i
Iteration k: Link i adapts as per (6), ∀i
for Relay r = 1 to N_r do
    if V_r(k) < −τ then
        j = arg max{η_j(k)} : w_j(k) = 1, j ∈ L_1^r;
        w_j(k+1) = 0;
    else if V_r(k) ≥ 0 then
        w_j(k+1) = 1, ∀j ∈ L_1^r;
    else
        w_i(k+1) = w_i(k), ∀j ∈ L_1^r
return to Iteration k = k + 1.
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IV. Network Adaptation Scheme

The distributed transmit power adaptations by UEs to solve (6) occur in time intervals denoted as $k \in \{1, 2, ..\}$. Each node has transmitter-side knowledge of the current effective interference $E_i(k)$ on its link. The determination of w_i for each link i is illustrated in Algorithm 1. Initially, each link sets out to maximize its data rate (i.e.

$w_i(0) = 1$ for each link i). However, if there is congestion on a RS-to-BS link, defined as when $V_r(k) < -\tau$ where τ is a positive-valued threshold, some of the UE-to-RS links may instead be selected to switch to power minimization mode to relieve the traffic load on the uplink backhaul. In each iteration, the selected link j is the one that has the highest instantaneous data rate. This selection continues iteratively until $V_r(k) \geq -\tau$ or until all UE-to-RS links go into power minimization mode.

Thus, as per Algorithm 1, a UE-to-RS link $j \in \mathcal{L}_1$ may either maximize its data rate or minimize its transmit power, whereas the power control objective of link $i \in \mathcal{L}_2 \cup \mathcal{L}_3$ (i.e. direct links to the BS) is always to maximize its data rate subject to the constraints in (6).

A. Marginal power efficiency-based adaptation

Given $w_i(k+1) = 1$, the objective in (6) becomes $\underbrace{\max}_{P_i} \eta_i$ for link i. Since transmit power is a limited resource, a UE needs to use it opportunistically and economically for time-varying wireless channels. For a fixed effective interference, increasing transmit power provides diminishing gains in capacity. To balance the marginal improvement in capacity (i.e. a node's utility) with the cost of increasing transmit power, a node may set the additional capacity obtained per unit power (i.e. marginal power efficiency) to a desirable threshold such that $\frac{\partial \eta_i}{\partial P_i} = \mu_i$:

$$\begin{aligned} \frac{\partial \eta_i}{\partial P_i} &= \frac{\partial}{\partial P_i} W \cdot \log_2 \left(1 + \frac{P_i}{E_i(k)}\right) \\ &= \frac{W}{\ln(2)(E_i(k) + P_i)} \end{aligned} \tag{7}$$

which is manipulated as

$$P_i + E_i(k) = \frac{W}{\ln(2)\mu_i} \tag{8}$$

Link i can be calibrated to transmit at full power if its effective interference tends to zero ($E_i \to 0$). Thus, the allocation becomes $P_i + E_i(k) = P_{\max,i}$ when we set $\mu_i = \frac{W}{\ln(2)P_{\max,i}}$. Note that this results in the *water-filling* for distributed power allocation over a single time-varying channel but without a constraint on the average transmit power as is the case in [16]. To achieve a minimal requisite SINR β, the following condition must hold:

$$\begin{aligned} \gamma_i &= \frac{P_i}{E_i(k)} = \frac{P_{\max,i}}{E_i(k)} - 1 \geq \beta \\ E_i(k) &\leq \frac{P_{\max,i}}{\beta + 1}. \end{aligned} \tag{9}$$

As in [13][16], the response to harsh channel conditions is for the link to delay its transmission until E_i falls below a threshold. Thus, the marginal power efficiency metric is appropriate for data services when always achieving a minimum target SINR is not a strict requirement. Given $w_i(k+1) = 1$ the transmit power of link i is updated under MPEA as:

$$P_i(k+1) = \begin{cases} P_{\max,i} - E_i(k) \text{ , if } E_i(k) \leq \frac{P_{\max,i}}{\beta+1} \\ 0\text{, otherwise.} \end{cases} \tag{10}$$

The corresponding effective interference in vector notation is

$$\mathbf{E}(k) = \mathbf{D} + \mathbf{F}\mathbf{P}(k). \tag{11}$$

Next we establish a sufficient condition for all links to achieve the minimum SINR under MPEA. The condition corresponds to a constraint on the maximum interference in the network denoted as $\mathbf{E}_{\max} = \mathbf{F}\mathbf{P}_{\max} + \mathbf{D}$.

Theorem IV.1. *If $(\beta+1)\mathbf{D} \leq [\mathbf{I} - (\beta+1)\mathbf{F}]\mathbf{P}_{\max}$ then all links under MPEA, regardless of the initial transmit power vector, will achieve the minimum SINR β.*

Proof. As per (10), links transmit with non-zero power in iteration $k+1$ if their current effective interference is less than the cutoff threshold (i.e. $E_i(k) \leq \frac{P_{max,i}}{\beta+1}, \forall i$). We rearrange $(\beta+1)\mathbf{D} \leq [\mathbf{I} - (\beta+1)\mathbf{F}]\mathbf{P}_{\max}$ into $(\mathbf{F}\mathbf{P}_{\max} + \mathbf{D}) \leq \frac{\mathbf{P}_{\max}}{\beta+1}$. The effective interference is maximized ($\mathbf{E}_{\max} = \mathbf{F}\mathbf{P}_{\max} + \mathbf{D}$) when all links transmit at full power. Hence, given the inequality condition (i.e. $\mathbf{F}\mathbf{P}_{\max} + \mathbf{D} = \mathbf{E}_{\max} \leq \frac{\mathbf{P}_{\max}}{\beta+1}$), the maximum effective interference possible (LHS) is always less than $\mathbf{P}_{\max}/(\beta+1)$ (RHS). Since $\beta > 0$, $\mathbf{P}(k+1) = \mathbf{P}_{\max} - \mathbf{E}(k) > \mathbf{0}$ always and it also follows that the SINR of each link remains above β. ☐

Equivalently, we have $\mathbf{E}_{\max} \leq \frac{\mathbf{P}_{\max}}{\beta+1}$ from the above inequality.

B. Backhaul Congestion

When $V_r(k) < -\tau$, then the relay indicates congestion to the UE-to-RS link j that currently has the highest data rate among the UE-to-RS links (see Algorithm 1). Then $w_j(k+1) = 0$, and link j switches to the objective $\underbrace{\max}_{P_j}(P_{\max,j} - P_j)$ as per (6). Its transmit power is adjusted as follows:

$$\begin{aligned} P_j(k+1) &= \min\left(P_{\max,j}, \beta E_j(k)\right) \\ &= \min\left(P_{\max,j}, \frac{\beta}{\gamma_j(k)} P_j(k)\right) \end{aligned} \tag{12}$$

which is the well-known Foschini-Miljanic (FM) algorithm [1] to meet the target SINR β [8]. Until the backhaul congestion is alleviated, such that that $V_r > -\tau$, an additional UE-to-RS link will switch from MPEA to the FM algorithm at each iteration. Our current approach is unlike [17], where all UE-to-RS links simultaneously reduce their transmit power if the RS-to-BS link suffers congestion. If the backhaul capacity becomes sufficient (i.e. $V_r(k) \geq 0$), then all UE-to-RS links to relay r may again adapt under MPEA. In the intermediate case, if $-\tau \leq V_r(k) < 0$, the adaptation objective of each link $j \in \mathcal{L}_1$ remains unchanged (i.e. $w_j(k+1) = w_j(k)$).

V. Convergence Conditions

We denote the power adaptation mode of links under MH-MPEA as an $L \times L$ diagonal matrix $\mathbf{W}(k+1) =$

Algorithm 2: Power control updates for iteration k.

for *UE* $i = 1$ *to* n **do**
  if $w_i(k) = 0$ **then**
    $P_i(k+1) = \min\left(P_{\max,i}, \beta E_i(k)\right)$;
  else if $w_i(k) = 1$ *and* $E_i(k) \leq \frac{P_{\max,i}}{\beta+1}$ **then**
    $P_i(k+1) = P_{\max,i} - E_i(k)$;
  else if $w_i(k) = 1$ *and* $E_i(k) > \frac{P_{\max,i}}{\beta+1}$ **then**
    $P_i(k+1) = 0$.

$diag[w_1(k+1), \cdots, w_L(k+1)]$. If the inequality condition in Theorem IV.1 holds, then the distributed power allocation for the optimization in (6) will become:

$$\begin{aligned}\mathbf{P}(k+1) &= [\mathbf{I} - \mathbf{W}(k+1)][\mathbf{P}_{\max} - \mathbf{E}(k)] \\ &\quad + \mathbf{W}(k+1)\beta\mathbf{E}(k) \\ &= \mathbf{Y} + \mathbf{X}\mathbf{P}(k)\end{aligned} \tag{13}$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{Y} = \beta\mathbf{W}(k+1)\mathbf{D} + [\mathbf{I} - \mathbf{W}(k+1)][\mathbf{P}_{\max} - \mathbf{D}]$ and $\mathbf{X} = \beta\mathbf{W}(k+1)\mathbf{F} - [\mathbf{I} - \mathbf{W}(k+1)]\mathbf{F}$.

Lemma V.1. *If* $\mathbf{E}_{\max} \leq \frac{\mathbf{P}_{\max}}{\beta+1}$, *then the spectral radius (maximum absolute eigenvalue) of* $\mathbf{X}$ *is less than* 1.

Proof. We manipulate the given inequality to obtain

$$\begin{aligned}\mathbf{E}_{\max} &\leq \frac{\mathbf{P}_{\max}}{\beta+1} \\ (\beta+1)\left(\mathbf{D} + \mathbf{F}\mathbf{P}_{\max}\right) &\leq \mathbf{P}_{\max} \\ \bar{\mathbf{F}}\mathbf{P}_{\max} &< \mathbf{P}_{\max}\end{aligned} \tag{14}$$

where $\bar{\mathbf{F}} = (\beta+1)\mathbf{F}$. Multiplying the last inequality above by $\bar{\mathbf{F}}^k$, where $k \in \mathbb{N}$, we obtain $\bar{\mathbf{F}}^{k+1}\mathbf{P}_{\max} < \bar{\mathbf{F}}^k\mathbf{P}_{\max}$ which implies that $\bar{\mathbf{F}}^{k+1}\mathbf{P}_{\max} < \bar{\mathbf{F}}^k\mathbf{P}_{\max} < \bar{\mathbf{F}}^{k-1}\mathbf{P}_{\max} < \cdots < \bar{\mathbf{F}}\mathbf{P}_{\max} < \mathbf{P}_{\max}$. This indicates that the inequality chain $\lim_{k\to\infty} \bar{\mathbf{F}}^{k+1}\mathbf{P}_{\max}$ is an all-zeros vector. This is true if and only if the spectral radius of $\bar{\mathbf{F}}$ is less than one (and thus that of $\mathbf{F}$ less than $\frac{1}{\beta+1}$) [18, pg. 618]. The non-diagonal matrix elements of row i in $\mathbf{X}$ are either the same as those of $\mathbf{F}$ (if $\mathbf{W}(k+1)(i,i) = 0$) or they are scaled by a factor of β (if $\mathbf{W}(k+1)(i,i) = 1$). The spectral radius of $\mathbf{X}$ is thus also bounded by $\frac{\beta}{\beta+1} < 1$ [18, pg. 498]. □

Next we show convergence from matrix series expansion. This is in contrast to the convergence analysis in [6–9], which is based on the *two-sided scalability* property.

Theorem V.2. *Given a high enough* τ *and* $\mathbf{E}_{\max} \leq \frac{\mathbf{P}_{\max}}{\beta+1}$, *then* $[\mathbf{I} - \mathbf{X}]^{-1}\mathbf{Y}$ *represents a fixed point solution to* (6) *for all links.*

Proof. Setting a high enough τ, indicating an acceptable congestion level, ensures that $V_r(k) > -\tau$ at each relay. Thus, for some value of τ, not all UE-to-RS links will switch to the FM algorithm and the matrix $\mathbf{W}(k+1)$ will thus become constant. Thus, given Theorem IV.1 and Lemma V.1, the transmit power vector converges to:

$$\begin{aligned}\mathbf{P}(k+1) &= \mathbf{X} + \mathbf{Y}\left(\mathbf{X} + \mathbf{Y}\left(\mathbf{Y} + \mathbf{X}\left(\cdots\mathbf{P}(0)\right)\right)\right) \\ &= [\mathbf{I} + \mathbf{X} + \mathbf{X}^2 + \cdots]\mathbf{Y} + \mathbf{X}^k\mathbf{P}(0) \\ \lim_{k\to\infty} \mathbf{P}(k+1) &= \mathbf{P}^* = [\mathbf{I} - \mathbf{X}]^{-1}\mathbf{Y}\end{aligned} \tag{15}$$

□

In practical wireless systems, estimation error will render transmitter-side knowledge about channel gain to its receiver imperfect [19]. The transmit power updates will thus be based on inaccurate effective interference values. Given the estimated gain $\widehat{g_{i,i}}$ for link i, the effective interference in (2) becomes:

$$\widehat{E_i}(k) = \frac{n_o + \sum_{j\neq i, j\in\mathcal{F}(f)} g_{j,i}P_j(k)}{\widehat{g_{i,i}}},$$

Essentially, the estimation error will project $\mathbf{E}(k)$ into

$$\widehat{\mathbf{E}}(k) = \widehat{\mathbf{D}} + \widehat{\mathbf{F}}\mathbf{P}(k),$$

where $\widehat{\mathbf{D}}(i) = n_o/\widehat{g_{i,i}}$ and $\widehat{\mathbf{F}}\mathbf{P}(k)$ is the received interference vector adjusted for the estimated transmitter-receiver gains on the links. Consequently, corresponding to the imperfect channel knowledge, if the inequality $\mathbf{E}^{'}_{\max} = \widehat{\mathbf{F}}\mathbf{P}_{\max} + \widehat{\mathbf{D}} < \frac{P_{\max}}{\beta+1}$ is satisfied then the results in Lemma V.1 and Theorem V.2 would still hold. Therefore, under MH-MPEA, nodes would converge to (15) with the values in $\mathbf{X}$ and $\mathbf{Y}$ adjusted for estimation error.

VI. FAIRNESS

Fairness is an important criterion in wireless network resource allocation [20, 21]. Note that the inequality condition $\mathbf{E}_{\max} \leq \frac{\mathbf{P}_{\max}}{\beta+1}$ may not always be met and some links may be unable to achieve an SINR of β under MPEA. Conversely, for fairness it is sufficient to show that all links can achieve a minimum SINR of β corresponding to some transmit power vector $\mathbf{P}^* = [P_1^*, P_2^*, \cdots, P_L^*]$, meeting the maximum power constraint,[11][12] (i.e. *feasibility*).

Theorem VI.1. *The sufficient and necessary condition for system feasibility is that the spectral radius of* $\mathbf{F}$ *be less than* $\frac{1}{\beta}$ *and that there be a transmit power vector* $\mathbf{P}^* = \beta[\mathbf{I} - \beta\mathbf{F}]^{-1}\mathbf{D}$ *such that* $\mathbf{P}^* \leq \mathbf{P}_{\max}$.

Proof. See [11][12] for details. □

It has been shown that if a feasible system exists then the FM power will be able to achieve the target SINR β for all links with minimum aggregate power [12].

The region of particular interest is when the system is feasible but some links fail to realize the minimum acceptable SINR when they use MPEA. When system throughput is preferred over instantaneous fairness, having a few links that are unable to achieve a minimal SINR may be justified if the resultant increase in the network's aggregate end-to-end data rate under MPEA is sufficiently high. For the feasible region, we compute the *throughput gain* as the expected value $\mathbf{E}\left[\frac{\eta_N^{''} - \eta_N^{'}}{\eta_N^{'}}\right]$, where $\eta_N^{''}$ is the aggregate end-to-end data rate of the network under our proposed scheme while $\eta_N^{'}$ is the corresponding performance under the FM scheme. The *proportion of links* with an SINR less than β can be defined as the expected value $\mathbf{E}[\frac{L_{IF}}{L}]$ where L_{IF} is the number of links unable to achieve β. We use this expectation to assess relative fairness under the scheme. Moreover, under MH-MPEA, UE-to-relay links with the highest current data rate switch to the FM power control if the relay backhaul is congested. This mechanism improves fairness between different UEs without diminishing system throughput by allowing starving flows to benefit from interference and congestion reduction by advantaged flows.

VII. Simulation Results

We consider a multihop network with $N_r = 3$ relays separated by 120^o and at a distance of 2000 m from the BS. We assume that the link gains are of the form $g_{j,i} = k_{j,i} d_{j,i}^{-\alpha}$, where α is the path loss exponent, $k_{j,i}$ and $d_{j,i}$ are the exponential fading gain (due to Rayleigh block fading) and the distance between the transmitter of link j and the receiver of link i respectively. The gains remain constant for 15 power control iterations. We have $\alpha = 3.0$ for all RS-to-BS links and $\alpha = 3.6$ for all UE links. The maximum transmit power of a UE is 0.5 Watt, that of RS-to-BS links is 3.0 Watts, $\beta = 1.59$ (2 dB), $W = 1$ MHz and $n_o = -120$ dBW.

We first verify (15) for the sample network as shown in Fig. 1. For this network, we generate random fading coefficients for all links such that $\mathbf{P}_{\max} \geq \frac{\mathbf{E}_{\max}}{\beta+\mathbf{1}}$. All links have a randomly selected initial transmit power level and adapt under MH-MPEA. With $w_1(k+1) = 0$ for UE-to-RS link A and $w_j(k+1) = 1$ for remaining links in Fig 1, the transmit powers converge to the corresponding (15) as shown in Fig. 2(a). Next we simulate the impact of the number of UEs n and how closely these UEs are clustered around the Points of Access (i.e. one BS and three RS) on the system's aggregate end-to-end data rate. The n UEs in the cell are randomly distributed within circular regions around the Points of Access according to a uniform distribution. The radius of the circular region around each of the four Points of Access is R_L m. For performance comparison, we benchmark against two schemes: (1) all links use the FM algorithm with target SINR β and (2) each link uses the OPC algorithm. The OPC algorithm has been adopted in [6–9]. Under OPC, each link i adapts as:

$$P_i(k+1) = \min\left(P_{\max,i}, \frac{\nu}{E_i(k)}\right) \tag{16}$$

to maintain a constant *Signal-to-Interference-Product* (SIP $\nu = P_i \cdot E_i$) [6]. For the simulation trials, we plot the system throughput performance by averaging the aggregate end-to-end data rate η_N. In Fig. 2(b) and Fig. 3 we observe that MPEA provides better performance than both FM and OPC algorithms (with different values of ν). The performance improvement is more pronounced when n is large or when R_L is small (i.e. UEs are clustered closely around their Point of Access). If the UEs are dispersed over a smaller region, the mutual interference between co-channel links becomes smaller

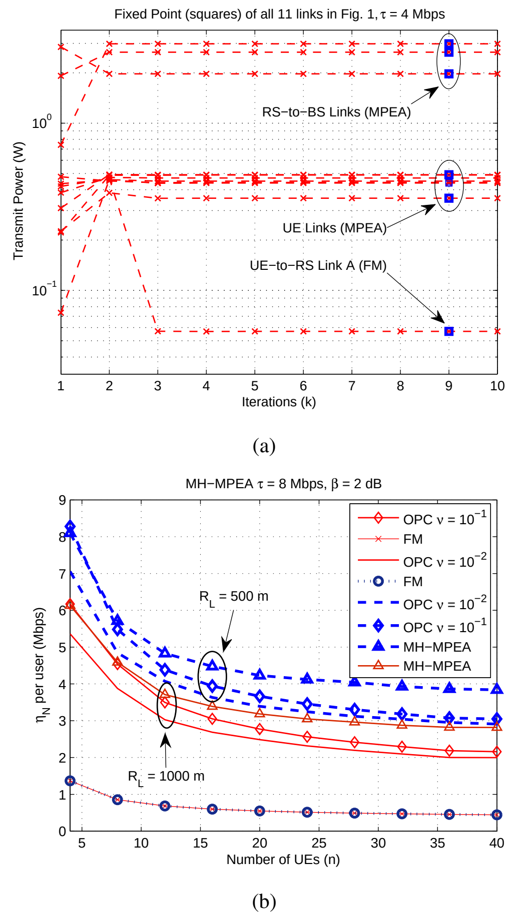
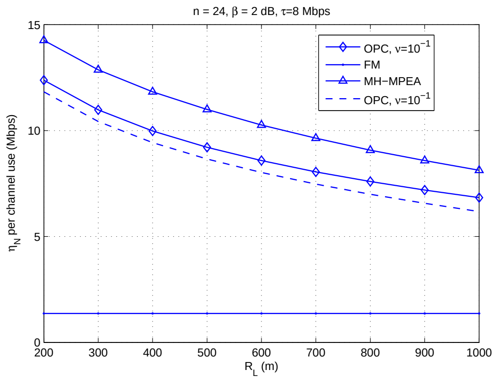


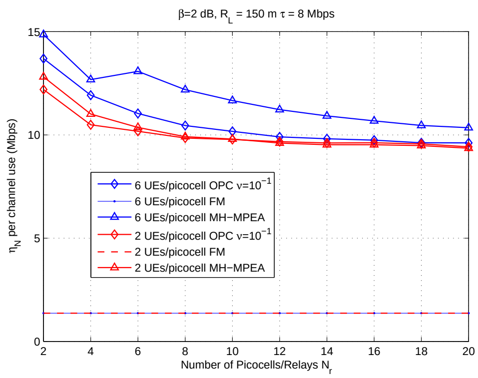
Fig. 2: (a) Random initial transmit power levels of links in the sample network converge to the vector in (15) (superimposed at $k = 9$), which is a solution of (6). (b) MH-MPEA enables better system throughput performance than both the FM and OPC algorithms with increasing n. At $v = 10^{-1}$, links are almost always transmitting at maximum power under OPC.

and correspondingly the system throughput is higher. In Fig. 3(b) we consider the case when a large number of relays are randomly and uniformly located within the cell in a 2 km × 2 km square region and the UEs are clustered close to them. The 2-hop network could resemble the situation when picocells with wireless backhaul links coexist within a macro-cellular system [22]. We again observe performance improvement under MH-MPEA, as compared to other schemes.

Next we compare the system throughput against fairness for MH-MPEA for both the multihop scenario and with only UE nodes in the network. In the single-hop (no relay) scenario, all UEs communicate directly with the BS and lie within a radius R_L m around the BS (i.e. MH-MPEA becomes an all-MPEA scheme). Considering the simulation trials when the system is feasible, in Fig. 4(a) we produce scatter plots of the sample mean of $\left[\frac{\eta_N^{''}-\eta_N^{'}}{\eta_N^{'}}\right]$ (throughput gain) against the sample mean of $[\frac{L_{IF}}{L}]$ (proportion of infeasible links).

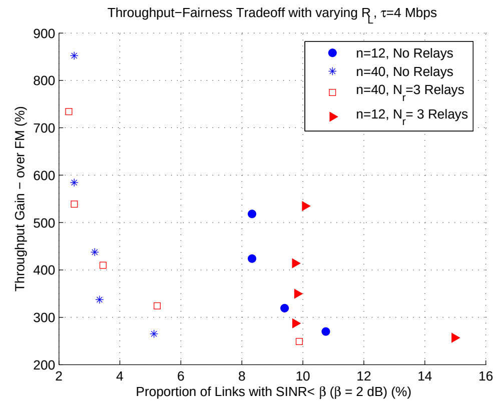


(a)

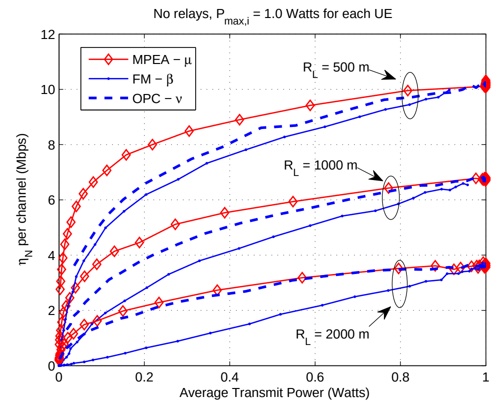


(b)

Fig. 3: (a) MH-MPEA has the best system throughput performance over the range of cluster radius R_L.(b) With increasing number of relays that have UEs clustered close to them, the multihop network approximates picocell networks overlaid within a macro-cell.



(a)



(b)

Fig. 4: (a) Note the significant throughput gains under MH-MPEA; the relative decrease in the *fraction* of infeasible links with more UEs implies that these gains are robust even with high traffic volume. (b) The average data rate achieved per link is higher for MPEA than that by either OPC or FM algorithms for the same average transmit powers.

The scatter plot is derived by varying R_L from 200m to 1000m for both the multihop and single-hop scenarios. We observe that the throughput gains are at least 200% whereas a maximum of 15% of the links are unable to meet the minimum target SINR. Also note that MH-MPEA is adept at bringing substantial throughput gains as the fairness penalty (i.e. *proportion* of links that are infeasible) becomes relatively smaller with a larger n.

Finally in Fig. 4(b), we compare the average data rate of a link with MPEA against FM and OPC algorithms in terms of average power use in a network with only UE nodes (i.e. no relays) and with $P_{\max,i} = 1.0$ Watts for each node. For the FM algorithm, the curve is obtained by adjusting the target SINR over $\beta \in [-10, 30]$ dB. For MPEA we adjust the value of marginal power efficiency as defined in (8) over $\mu \in [10^{-3}, 10^3]$ whereas for the OPC algorithm, its curve is obtained by adjusting the target SIP over $\nu \in [10^{-4}, 10^0]$. The larger the value of ν or β, the more power is consumed and higher the throughput. We can observe that over the range of parameters, the achieved data rates are larger for MPEA than for FM and OPC algorithms.

VIII. Conclusions

We have proposed a marginal power efficiency-based power control scheme that can be used in conjunction with the FM algorithm for cellular systems, taking into account network-wide conditions to regulate each link's transmit power. We have derived a closed-form expression for a unique fixed point of the transmit powers under our adaptation approach and have found a significant benefit to overall network performance in terms of both system throughput and relative fairness. We observe that the proposed adaptation scheme is particularly suitable for LTE-Advanced networks that will deploy relays to improve coverage.

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