This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

Autonomous Sensing Order Selection Strategies Exploiting Channel Access Information

Zaheer Khan, Janne J. Lehtomäki, *Member, IEEE,* Luiz A. DaSilva, *Senior Member, IEEE,* Matti Latva-aho, *Senior Member, IEEE*

Abstract

We design an efficient sensing order selection strategy for a distributed cognitive radio (CR) network, where two or more autonomous CRs sense the channels sequentially (in some sensing order) for spectrum opportunities. We are particularly interested in the case where CRs with false alarms autonomously select the sensing orders in which they visit channels, without coordination from a centralized entity. We propose an adaptive persistent sensing order selection strategy and show that this strategy converges and reduces the likelihood of collisions among the autonomous CRs as compared to a random selection of sensing orders. We also show that, when the number of CRs is less than or equal to the number of channels, the proposed strategy enables the CRs to converge to collision-free channel sensing orders. The proposed adaptive persistent strategy also reduces the expected time of arrival at collision-free sensing orders as compared to the *randomize after every collision* strategy, in which a CR, upon colliding, randomly selects a new sensing order.

Index Terms

Autonomous cognitive radios, adaptation, multichannel cognitive radio networks, opportunistic spectrum access.

I. INTRODUCTION

Cognitive radio networks are envisioned to utilize the licensed frequency spectrum more efficiently through opportunistic access to (temporarily) unused spectrum bands. Among different opportunistic spectrum access (OSA) schemes, sensing-based OSA is widely investigated because it does not require the licensed (primary) users to alter their existing hardware or behavior [1]. In sensing-based OSA, cognitive

Z. Khan is a Ph.D. candidate at the Centre for Wireless Communications (CWC), University of Oulu, Finland.

J. J. Lehtomäki is with the Centre for Wireless Communications (CWC), University of Oulu, Finland.

L. A. DaSilva is the Stokes Professor at CTVR, the Telecommunications Research Centre, Trinity College Dublin, Ireland and is also a Professor at Virginia Tech, Blacksburg, USA.

Matti Latva-aho is a Professor at Centre for Wireless Communications (CWC), University of Oulu, Finland.

This work was funded by the Academy of Finland and National Technology Agency of Finland (Tekes).

Digital Object Indentifier 10.1109/TMC.2011.257

radios (CRs) monitor the environment to reliably detect the primary user signals and operate whenever the band is empty. In practice, detection of primary users may rely on a combination of sensing and the use of geolocation spectrum occupancy databases [2].

When multiple frequency bands (channels) are available for opportunistic transmissions, time-slotted multiple access is widely considered [3]–[8]. The first portion of each time slot is used by CRs for spectrum sensing, and the second portion is used to access the free channel, if one is found. However, when multiple autonomous CRs have to search multiple channels for spectrum opportunities, these radios face competition from one another to access the channel. For instance, if in a given slot a particular channel is simultaneously sensed free by two or more autonomous CRs and more than one of them decide to transmit on the channel, then a collision occurs. In this context, the channel sensing order $\mathbb{P}$, i.e., the order in which radios competing for the channels visit those channels, will affect their probability of successful access. This paper investigates how CRs can autonomously select channel sensing orders so as to minimize the likelihood of collisions with other CRs also searching for channels to be utilized opportunistically.

In a given time slot, CRs searching for a channel face one of the following outcomes: success, collision, or no transmission (when all channels sensed by that CR were found busy). Can CRs then use the history of past outcomes to find a way to autonomously arrive at collision-free sensing orders? If the answer is yes, can we design efficient sensing order selection strategies that maximize the throughput of the distributed CR network?

The contributions of this paper are four-fold:

- We propose and evaluate an adaptive persistent sensing order selection strategy that enables the CRs to reduce the likelihood of collisions with one another, as compared to a random selection of sensing orders. Adaptations are in the autonomous choice, by CRs, of the channel sensing order $\mathbb{P}$. We find that the proposed strategy converges to collision-free sensing orders without requiring any coordination among CRs (provided that the number of CRs is less than or equal to the number of potentially available channels). Collision-free sensing orders are those in which two or more CRs never simultaneously sense the same channels and therefore never collide with one another. We also find that the adaptive persistent strategy reduces the expected time of arrival at collision-free sensing orders as compared to the *randomize after every collision* strategy, in which a CR, upon colliding, randomly selects a new channel sensing order.
- We explore the impact of imperfect information, i.e., the effects of false alarms and channel errors,

on adaptation decisions. We also investigate the impact of different primary user (PU) channel occupancy models and find that the proposed adaptive strategy is not strongly affected by PU behavior.

- We show that, when adaptation is employed, there is an increase in the average number of successful transmissions in the network when the CRs select sensing orders from a predefined Latin Square, as compared to when they select sensing orders from the space of all permutations of N channels. A Latin Square is an N by N matrix of N channel indices in which every channel index occurs exactly once in each row and column of the matrix [9], [10].

- We derive closed-form expressions for the probability of success (the probability that a given CR finds a channel free) for CRs competing for opportunistic use of channels. We derive these expressions for a general number M of distributed CRs competing for these channels. In the literature, results for the probability of success for sequential-channel sensing policy typically do not consider distributed decisions without assumptions regarding a priori knowledge of PU activity statistics or knowledge of channel gains. Surprisingly, we find that when no adaptation is employed, a non-zero probability of false alarm can actually increase the probability that a CR successfully finds a channel to transmit in. This counter-intuitive result stems from a reduced number of collisions among the autonomous CRs, as further explained in Section V-A. To validate the closed-form expressions that were derived, we compare these results to results obtained via simulations.

The remainder of the paper is organized as follows. Section II summarizes some of the relevant literature on the problem of multichannel sensing and allocation for CRs. Section III presents the system setup, while Section IV presents and analyzes our proposed adaptive persistent sensing order selection strategy. In Section V we compare the proposed sensing order selection strategy to related strategies proposed in other works, and Section VI summarizes our main conclusions and outlines some directions for future work.

II. Related Work

In sensing-based OSA, the CRs are required to perform periodic spectrum sensing so that when a primary user becomes active in a channel, the CRs can vacate that channel [11]. In Fig. 1, a taxonomy of time slotted periodic spectrum sensing models consisting of two branches is presented: periodic sensing for a single potentially available primary user band and periodic sensing for multiple potentially available primary user bands. Under a single potential primary user band system, CRs are allowed to explore a single licensed spectrum band. The first portion of each time slot is used by CRs for sensing the licensed

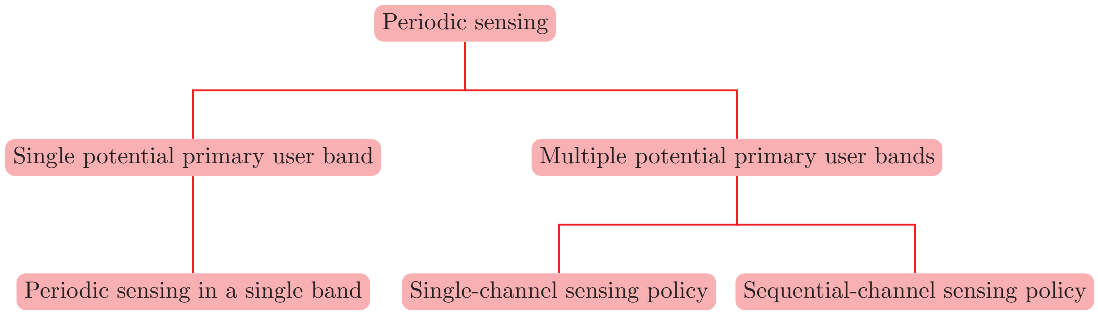


Fig. 1. Taxonomy of time slotted periodic sensing models for opportunistic spectrum access.

band, and the second portion is used to access the band, if it is free (as described by [12] and references therein).

In the other branch, periodic sensing for multiple potentially available primary user bands, CRs are allowed to explore multiple licensed spectrum bands [13]. From here, there are two broad categories of periodic sensing policies: CRs may employ a single-channel sensing policy or a sequential-channel sensing policy. Under a single-channel sensing policy, in any given time slot a CR first selects a channel to sense and transmits if that channel is free; otherwise, it stays silent for the entire duration of that time slot. The works in [6], [14]–[18] proposed distributed learning and allocation strategies for CRs employing a single-channel sensing policy. In particular, the works in [6] and [14] proposed a learning scheme that employs adaptive randomization based on feedback (occurrence of collisions) for the CRs to arrive at orthogonal channel selections. Under sequential-channel sensing, studied in this paper and also described in [5], [7], [19]–[21], a CR can sense more than one channel within the duration of a time slot. In this approach, two or more autonomous CRs sense the channels sequentially (in some sensing order) for spectrum opportunities.

Several optimal policies for the selection of channel sensing orders for the sequential channel sensing model are proposed in the literature. The works in [7], [19], [20] propose optimal policies for the selection of a channel sensing order $\mathbb{P}$ for a single CR with perfect sensing observations. Unlike [7], [19], [20], our work takes into account competition for channels among multiple CRs with false alarms. The selection of an optimal sensing order by a coordinator for a two-CR network is the topic of [4]. The coordinator determines the sensing orders to be adopted by the two CRs based on the estimated channel availability statistics and announces these sensing orders to the two CRs. While the two-user CR network with a coordinator is simple to implement, in practise a network comprising a large number of CRs would require significant signaling overhead to coordinate successful channel utilization. Moreover, in some

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

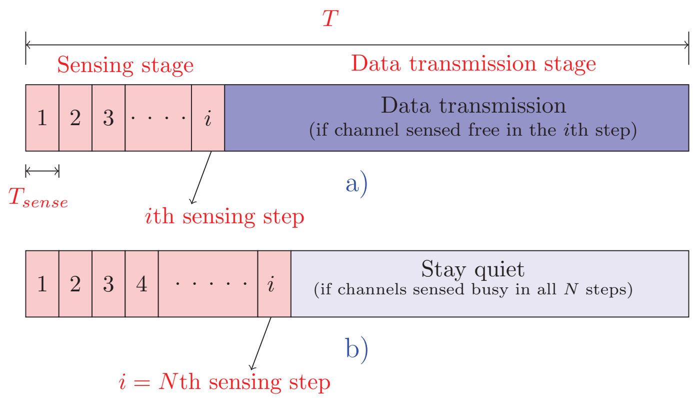


Fig. 2. Time slot structure with sensing and data transmission stages. a) If a CR finds a channel free in its ith sensing step, it transmits in that channel until the end of the slot. b) If in all sensing steps channels are sensed to be busy then the CR stays silent for the entire duration of that time slot.

practical scenarios, the CRs may be owned and managed by different service providers, requiring a sensing order selection strategy that does not rely on a common coordinator. A channel sensing order policy for distributed CR networks is proposed in [5]. However, this work assumes that CRs have knowledge of the gains for each channel. Based on this assumption, [5] proposes that each CR should sense channels in descending order of their achievable rates and should transmit in the first channel that is sensed free. Unlike [5], our work does not assume knowledge of the channel gains. Moreover, in contrast with [5], we propose and analyze adaptive sensing order selection strategies to minimize conflicts among CRs when accessing available channels.

The works in [22]–[24] propose learning-based medium access control (MAC) techniques that discover collision-free schedules. However, since the proposed techniques are designed for traditional Time Division Multiple Access (TDMA) based MACs, these schemes do not consider the possibility that a channel may not be available (due to the presence of a primary user) and that the transmitter must perform sensing before transmission to determine which channels are available. Therefore, the techniques proposed in [22]–[24] cannot be applied straightforwardly to distributed CR networks.

The radio rendezvous problem studied by us in [13] is, in a sense, the dual of the problem we study here. While [13] proposes the use of non-orthogonal sequences to increase the probability of rendezvous, in the present work we devise methods that increase the likelihood that CRs will independently arrive at collision-free sensing orders.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

III. System Model

We consider a distributed cognitive radio (CR) network of M cognitive transmitter/receiver pairs and a set $\mathbf{N} = \{1, 2, ..., N\}$ of channels. A CR is allowed to make use of one of these channels when the channel is not occupied by a primary user. The primary users and CRs are both assumed to use a time slotted system, and each primary user is either present for the entire time slot, or absent for the entire time slot [3], [6], [15]. Due to hardware constraints, at any given time each CR can either sense or transmit, but not both. Also, each CR can sense only one channel at a time. To protect transmissions by the incumbent, the detection probability $(P_{d,i})$ of an autonomous CR is fixed at a desired target value, $P_{d,i} = P_d$, for all $i \in \mathbf{M}$. In practise, P_d is required to be close to 1 [25]. In the literature this is defined as constant detection rate (CDR) requirement [26]. For a fixed target detection probability, false alarm of an autonomous CR is a variable. In this paper, for simplicity, we ignore the effect of the missed detection errors (since the desired detection probability is considered to be close to 1) and consider the effect of varying probabilities of false alarm.

We investigate the proposed methods under two different PU activity models: 1) In each channel, the probability of the PU being present in a given time slot is θ_i, $i \in \mathbf{N}$. In this model, for each channel, the PU activity in a time slot is independent of the PU activity in other time slots and is also independent of the PU activity in other channels; this (i.i.d.) model of PU channel occupancy is also adopted by [3], [5], [14]. 2) The second model considers correlation in channel occupancy by a PU in consecutive time slots. In this model, the state of each channel is described by a two-state Markov chain, with α_i indicating the transition probability for the ith channel from PU-occupied to PU-free and β_i indicating the transition probability from PU-free to PU-occupied. This PU activity model is also adopted in [27].

The selection of the channel for opportunistic transmission is determined as follows: The CRs use the beginning of each slot to sense the channels sequentially in some order $\mathbb{P}$ (based on their sensing order selection strategies, as explained in Sections IV and V) to find a channel that is free of PU (or other CR) activity. We refer to this as the sensing stage (see Fig. 2). The CR then accesses the first vacant channel it finds, if one exists. We refer to this as the data transmission stage. In this work the channel availability statistics are assumed to be unknown and CRs do not attempt to learn the channel availability statistics using the sensing samples. Therefore, all potentially available channels are treated equally by the autonomous CRs.

The sensing stage in each slot is divided into a number of *sensing steps*. Each sensing step is used by

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

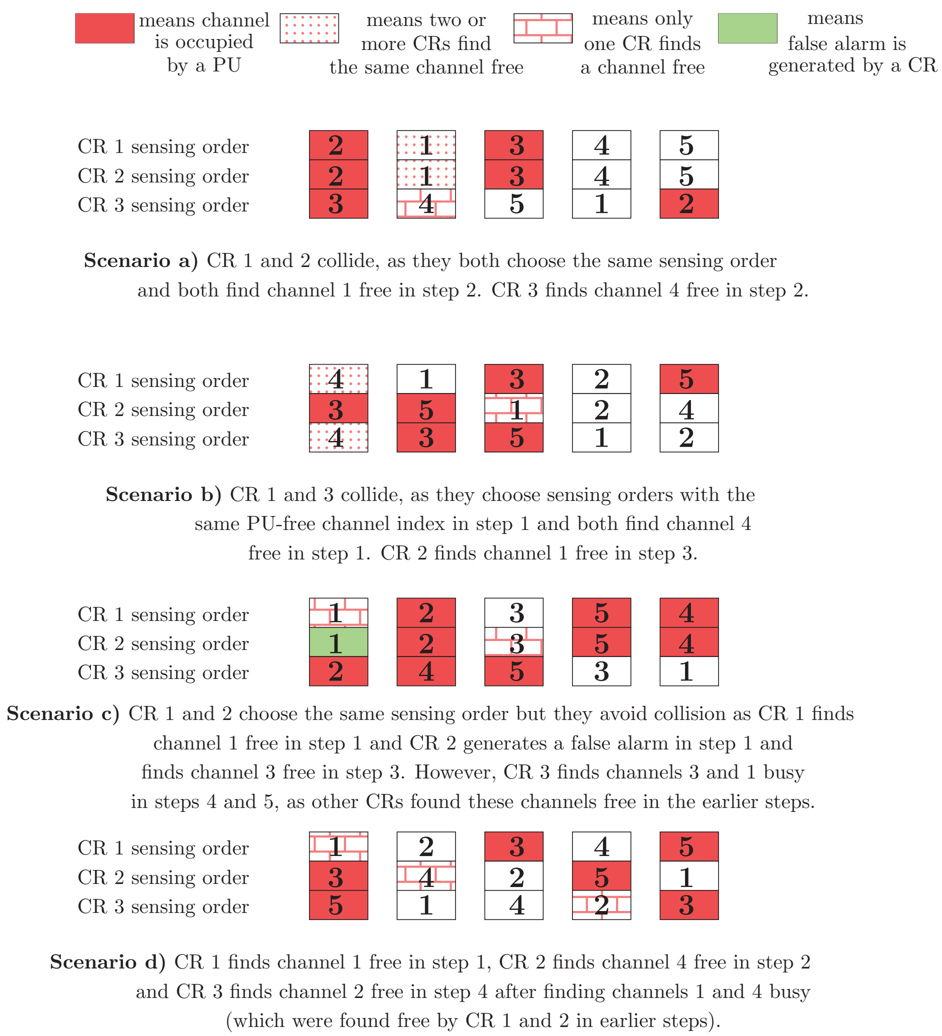


Fig. 3. Different scenarios for sequential channel sensing using sensing orders.

a CR to sense a different channel. If a CR finds a channel free in its ith sensing step, it transmits in that channel. However, if in all sensing steps channels are found to be busy, then the CR stays silent for the remaining duration of that time slot (see Fig. 2). When a free channel is found in the ith sensing step, the durations of the sensing stage and data transmission stage are iT_{sense} and $T - iT_{sense}$, respectively, where T_{sense} is the time required to sense each channel, T is the total duration of each slot and $T >> T_{sense}$. When multiple autonomous CRs search multiple potentially available channels for spectrum opportunities, then from an individual CR perspective one of the following three events will happen in each sensing step: 1) The CR visits a given channel and is the only one to find it free and transmit; the CR then has the channel for itself for the remainder of the time slot; 2) The CR visits a given channel, finds it occupied by the PU or by another CR, then it continues looking in the next sensing step; 3) The CR visits a given channel, finds it free and transmits, but so does at least one other CR; a collision occurs and the CR is

1	2	3	4
1	2	4	3
1	3	2	4
1	3	4	2
1	4	2	3
1	4	3	2
2	1	3	4
2	1	4	3
2	3	1	4
2	3	4	1
2	4	1	3
2	4	3	1

3	1	2	4
3	1	4	2
3	2	1	4
3	2	4	1
3	4	1	2
3	4	2	1
4	1	2	3
4	1	3	2
4	2	1	3
4	2	3	1
4	3	1	2
4	3	2	1

a)

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

b)

Fig. 4. Examples of sensing orders for $N = 4$ potentially available channels. In a), the space of all permutations of $N = 4$ channels is illustrated, while in b), a subset of that space (a Latin Square) is illustrated. A sensing order is a row of a Latin Square or a row of a permutation space of N channel indices.

not able to transmit until the next time slot, when it again will contend for a channel.

Note that a false alarm would have the effect of a CR thinking a channel is busy when it is in fact free of both PU and other CR activity. Figure 3 illustrates examples of different scenarios for sequential channel sensing using sensing orders.

Let X be a random variable representing the number of sensing steps within a time slot until a CR is successful in finding a channel free from PU and other CR activity (given that the CR is successful). With a constant time slot of duration T, the duration of successful data transmission in each slot is a function of X. Note that if N is larger than T/T_{sense}, then the CR does not have time to visit all channels within a time slot.

In this paper, we are interested in sensing order selection strategies that maximize the average number of successful transmissions in the distributed CR network under the PU protection constraint. Note that a successful transmission in a given slot means that a CR finds a channel free from PU activity and is the sole CR to transmit in that channel.

IV. Proposed Sensing Order Selection Strategy

When autonomous CRs have to search multiple potentially available channels for spectrum opportunities, they face competition from one another to access the channel. The end result of this competition is reduced CR throughput due to collisions among CRs that transmit in the same channel. We are interested in finding a way for distributed CRs to autonomously reach collision-free sensing orders. Collision-free sensing orders are those in which two or more CRs never simultaneously sense the same channels and therefore never collide with one another. In this section, we propose an adaptive persistent strategy that enables the CRs to reduce the likelihood of collisions. Moreover, the proposed adaptive strategy allow

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

γ-persistent strategy

1) Initialize $\mathbf{p} = [\frac{1}{|\mathbf{S}|}, \frac{1}{|\mathbf{S}|}, \ldots, \frac{1}{|\mathbf{S}|}]$ and set the binary flag and the success counter $b = SC = 0$.

2) Toss a weighted coin to select a sensing order, with p_i the probability of choosing sensing order i. Sense the channels sequentially in the order given in the selected sensing order.

3) One of three possibilities occurs:

a) *Successful transmission:* On a successful transmission using the current sensing order i, the CR updates p_i and p_j as $p_i = 1$ and $p_j = 0$, $\forall j \neq i$, i.e., it utilizes the same sensing order to visit the channels in the next slot. The CR then sets $SC = SC + 1$.

b) *CR finds all channels busy:* On using sensing order i in the current slot if all the channels visited by the CR are found to be currently occupied by either a PU or another CR, the CR updates p_i and p_j as $p_i = 1$ and $p_j = 0$, $\forall j \neq i$, i.e., it utilizes the same sensing order to visit the channels in the next slot. The CR then sets $b = 1$.

c) *CR collides with another CR:* On experiencing a collision in the current slot using sensing order i, the the CR updates p_i as

$$p_i = \begin{cases} 1/|\mathbf{S}| & \text{if } SC = 0 \text{ and } b = 1 \\ \gamma p_i & \text{otherwise} \end{cases}$$

and updates p_j as

$$p_j = \begin{cases} 1/|\mathbf{S}| & \text{if } SC = 0 \text{ and } b = 1 \\ \gamma p_j + \frac{1-\gamma}{|\mathbf{S}|-1} & \text{otherwise} \end{cases}$$

i.e., on experiencing a collision in the current slot the CR randomly selects a sensing order whenever $SC = 0$ and $b = 1$; otherwise the CR multiplicatively decreases the probability of picking sensing order i, redistributing the probability evenly across the other sensing orders. The CR then sets $SC = b = 0$.

4) Return to 2.

Fig. 5. γ-persistent strategy for sensing order selection.

the CRs in a distributed cognitive network to autonomously arrive at collision-free channel sensing orders (provided that the number of CRs is less than or equal to the number of potentially available channels).

Let $\mathbf{S}$ denote the set of available sensing orders. Note that the sensing order that a CR employs can either come from the space of all permutations of N channels (see Fig. 4a), or from some subset thereof. In this work, we propose an approach in which a sensing order comes from a common pre-defined Latin Square (LS), i.e., an N by N matrix of N channel indices in which every channel index occurs exactly once in each row and column of the matrix [9], [10]. An example of a Latin Square is illustrated in Fig. 4 (b). In Section V, we compare the performance of adaptive and non-adaptive sensing order selection strategies for two different scenarios: 1) Each CR selects its sensing orders from a common predefined Latin Square; 2) Each CR selects its sensing orders from the space of all permutations of N channels.

A. γ-persistent strategy

We propose a γ-*persistent* strategy, in which a CR uses a success counter (SC) and a binary flag b that track its successes and failures in using the current sensing order in prior time slots. The proposed strategy has a parameter γ, the persistence factor. We propose two different approaches for the use of γ: 1) An autonomous CR employs fixed values of $\gamma \in (0,1)$; 2) An autonomous CR employs $\gamma = 1-(\frac{1}{SC-log_2(P_{fa})})$ (which takes into consideration its false alarm probability and success counter (SC)). Note that the latter approach assumes that an autonomous CR can estimate its false alarm probability [12]. In the following subsections we will identify reasonable values for γ, when fixed values of γ are used, and will also explain the motivation behind the proposed γ as a function of false alarm probability and SC.

Let each CR maintains an $|\mathbf{S}|$-element probability vector $\mathbf{p}$ (i.e., all components are non-negative and add to 1). Let p_i represent the probability of selecting the ith sensing order (which may come from either the space of all permutations of N channels, or from a Latin Square). The γ-*persistent* strategy is described in Fig. 5. The detection of a collision is implemented in the following way. A CR infers that a collision has occurred whenever it fails to receive an acknowledgement (ACK) for a transmitted data frame.

In order to understand the motivation for the γ-persistent strategy, let us consider what happens when CRs select sensing orders from a common pre-defined Latin Square. Note that, using a Latin Square, $|\mathbf{S}| = N$ and two or more CRs will collide only if they select the same sensing order.

Rationale of the equations in step 3c) (Fig. 5) for updating p_i and p_j:

Step 3c) considers the possibility when a CR collides with another CR in the current time slot. On experiencing a collision in the current time slot the success counter SC and the binary flag b of the CR can be in one of the following states:

i) $SC > 0$ and $b = 0$; ii) $SC > 0$ and $b = 1$; iii) $SC = 0$ and $b = 0$; and iv) $SC = 0$ and $b = 1$

The rationale for the update of p_i and p_j after collision when $SC > 0$ and $b = 0$ or $b = 1$ is as follows:

An autonomous CR cannot determine on its own that the sensing order it has selected was not also selected by any other CR. However, a successful transmission $SC > 0$ indicates a high probability that the CR is the sole user of that sensing order in time slot n. After experiencing a collision, a CR persists with the sensing order i with probability γp_i, redistributing the probability $(1-\gamma p_i)$ evenly across the other sensing orders. This persistence improves the speed of convergence to collision-free sensing orders as successful CRs tend to stick with their sensing orders, reducing the number of CRs randomly selecting a sensing order.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

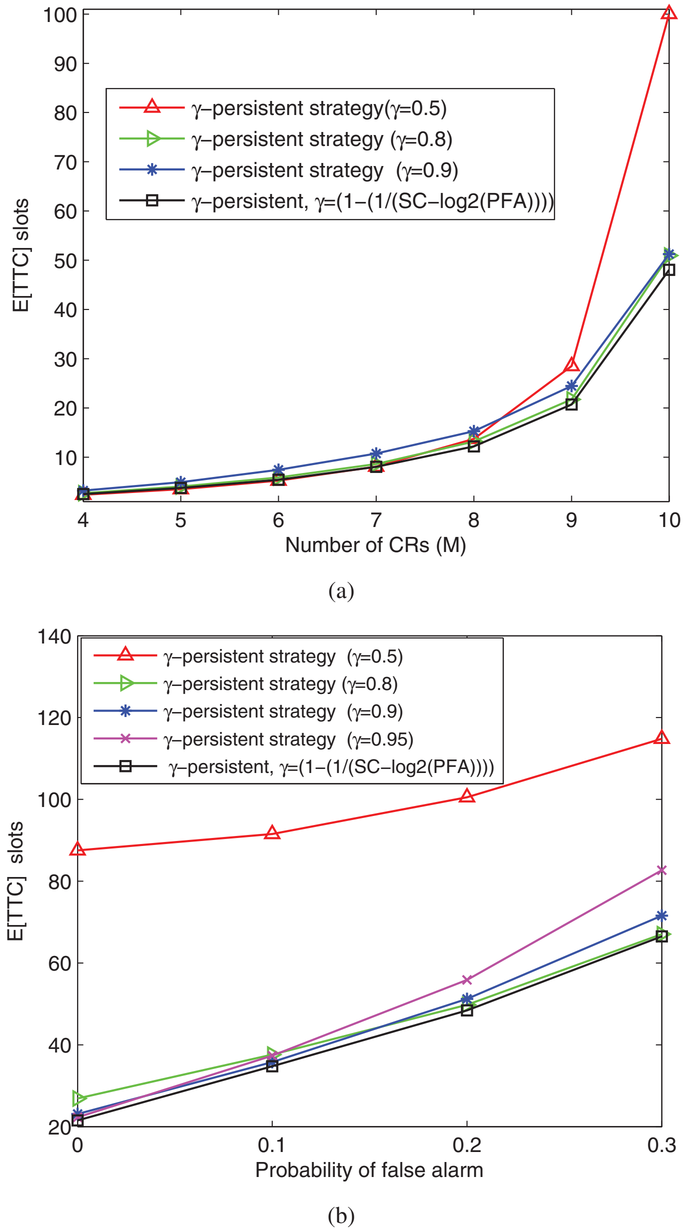


Fig. 6. E[TTC] as a function of: a) Number of CRs M (with false alarm probability of each CR set to 0.2); b) False alarm probability (with $M = 10$ CRs). The number of channels (N) is 10 and the probability of the PU being present in a given time slot (i.i.d model) is $\theta_i = 0.3$, $\forall i \in \mathbf{N}$; in all cases, selected sensing orders come from a Latin Square.

The rationale for the update of p_i and p_j after collision when $SC = 0$ and $b = 0$ is as follows:

The state $SC = 0$ and $b = 0$ indicates that, before experiencing a collision in the current time slot, the CR was neither successful nor found all channels busy using the sensing order i. Therefore, the CR decreases the probability of selecting that sensing order (as another CR may have been successful before and may stick with that sensing order) in the next time slot, and redistributing the probability $(1 - \gamma p_i)$ evenly across the other sensing orders.

The rationale for the update of p_i and p_j after collision when $SC = 0$ and $b = 1$ is as follows:

TABLE I
E[TTC] FOR THE γ-PERSISTENT STRATEGY UNDER I.I.D AND *Markov* PU OCCUPANCY MODELS, WHEN $N = M = 10$. FOR THE I.I.D PU OCCUPANCY MODEL $\theta_i = 0.3$ AND FOR THE MARKOV MODEL $\alpha_i = 0.3$, $\beta_i = 0.12857$ (SEE SECTION III), $\forall i \in \mathbf{N}$.

	$P_{fa} = 0$	$P_{fa} = 0.1$	$P_{fa} = 0.2$	$P_{fa} = 0.3$
	γ-persistent strategy ($\gamma = 0.5$)			
i.i.d	87.5344	91.5180	100.5057	114.8016
Markov	93.8481	95.1142	103.3093	113.9485
	γ-persistent strategy ($\gamma = 0.8$)			
i.i.d	26.8745	37.6155	49.7749	67.0764
Markov	29.5679	38.1646	51.5461	68.1112
	γ-persistent strategy ($\gamma = 0.9$)			
i.i.d	23.1055	35.7700	51.2426	71.5800
Markov	25.5543	36.3335	50.1692	72.0425
	γ-persistent strategy ($\gamma = 0.95$)			
i.i.d	22.2857	37.3654	55.8723	82.7012
Markov	24.9660	39.0157	56.4288	80.6743

$SC = 0$ and $b = 1$ means that using a sensing order i the CR k found all channels busy in time slot n. Since the CR k stays quiet, it cannot be sure whether it was the sole user of the sensing order in time slot n (as it may happen that one or more other autonomous CRs also selected the same sensing order and found all channels busy or they found a channel free and transmitted). Since the CR k cannot be sure whether it was the sole user of the sensing order in time slot n (since $SC = 0$), after a collision CR k will select a sensing order independently and randomly (with uniform probability).

The rationale for setting $SC = 0$ and $b = 0$ after a collision is as follows:

Let us assume that before experiencing a collision a CR was successful using a sensing order i, i.e., its success counter SC is set to greater than 0. After experiencing a collision, in the next time slot it may happen that the CR selects with probability p_j some other sensing order j, $j \neq i$. In this scenario if $SC > 0$ then the CR will incorrectly believe that it was successful using the sensing order j in the previous time slots. Similarly, if the binary flag b of the CR was set to 1 before collision and the CR selects with probability p_j some other sensing order j, $j \neq i$, then the CR will incorrectly believe that it was unable to find a free channel using the sensing order j in the previous time slots. To avoid this, the CR after experiencing a collision resets the values of SC and b to 0 (see step 3c of the γ-persistent strategy).

Choosing the persistence factor γ:

We propose two different approaches for the use of γ: 1) An autonomous CR employs fixed values of

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

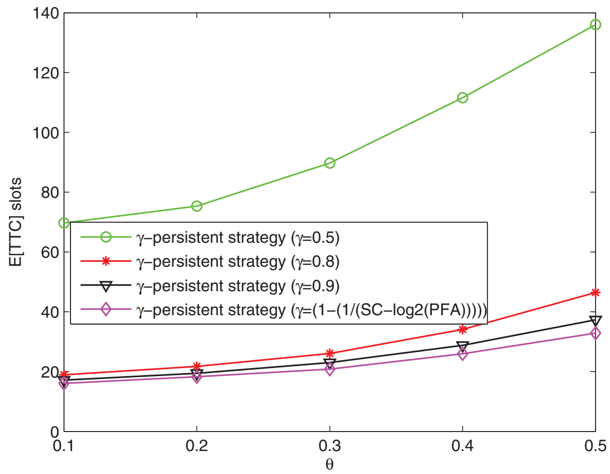


Fig. 7. E[TTC] as a function of varying the probability of the PU being present in a channel in a given time slot (i.i.d model), where θ_i, $\forall i \in \mathbf{N}$. The number of channels (N) is 10, the number of CRs is $M = 10$ and the false alarm probability of each CR is set to 0.

$\gamma \in (0,1)$; 2) An autonomous CR employs $\gamma = 1 - (\frac{1}{SC - log_2(P_{fa})})$. Note that by presenting γ as a function of the false alarm and the success counter (SC) the latter approach takes into consideration a CR's false alarm probability and also indirectly (through the SC) the number of active CRs in the distributed network. While we make no claims as to the optimality of the selected persistence factor γ for the second approach, we justify it as follows. Using the proposed $\gamma = 1 - (\frac{1}{SC - log_2(P_{fa})})$, a CR with low false alarm probability and a high number of successful transmissions (using a sensing order i), after experiencing a collision persists with that sensing order with high probability. However, if the CR has a high false alarm probability and a low number of successful transmissions using a sensing order i, after experiencing a collision it persists with that sensing order with low probability. In Figs. 6a and 6b, we plot the expected time to arrive at collision-free sensing orders (E[TTC]) of the proposed strategy as a function of the number of CRs M and the false alarm probability. The two figures show that when the proposed strategy employs γ as a function of false alarm probability and SC, it performs at least as well or better than the scenarios where fixed values of γ are employed.

In Table I, we list the expected TTC for the γ-persistent strategy (using fixed values of γ) under the i.i.d and Markov PU occupancy models; in all cases, selected sensing orders come from a Latin Square. The results in Table I also indicate that the expected TTC for the proposed adaptive strategy is not strongly affected by different PU occupancy models. Fig. 7 evaluates the effect of varying the values of $\theta_i = \theta$, $\forall i \in \mathbf{N}$, i.e., the probability of the PU being present in a channel, on the performance of the proposed scheme. It can be seen in Fig. 7 that E[TTC] increases as the value of θ increases. This is due to the

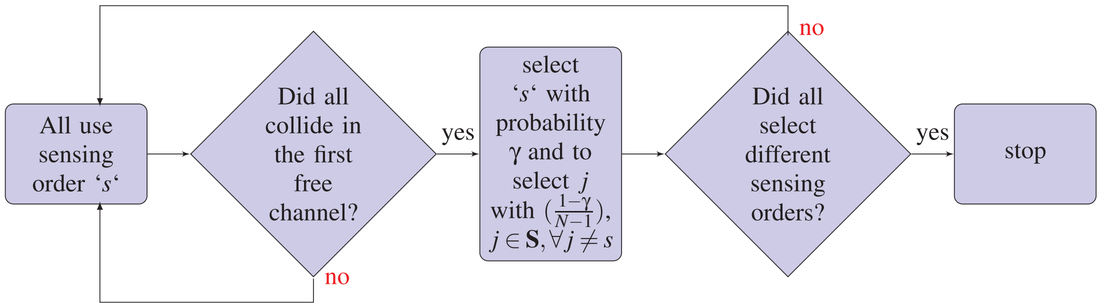


Fig. 8. Genie-aided modification.

fact that the proposed scheme utilizes feedback (occurrence of successful transmissions or collisions) to arrive at collision-free sensing orders. High values of θ reduce the chances of successful transmissions or collisions between CRs (as the number of available free channels is reduced), which in turn require more number of time slots to arrive at collision-free sensing orders.

B. Convergence of the γ-persistent strategy

In this subsection, we prove the convergence of the proposed γ-persistent strategy for the scenario when $N = M$ and $0 < P_{fa,i} < 0.5$. Note that when $N > M$ or $P_{fa,i} = 0$, or both, the convergence process is similar to the $N = M$ scenario but faster (see Figs. 6a and 6b).

Theorem 4.1: Under the γ-persistent strategy, when $N = M$ CRs select sensing orders from a common pre-defined Latin Square, then for any $\gamma \in (0,1)$, the network converges to collision-free sensing orders.

Proof: Let E represent the event that, in a given slot, the CRs autonomously arrive at collision-free sensing orders and P_E the probability of this event. To show that the proposed γ-persistent strategy converges, we consider a genie-aided modification of autonomous arrival at collision-free sensing orders. In the genie-aided modification each CR employs a common pre-defined sensing order (row) s of the Latin Square for sequential channel sensing. If all the M CRs collide in the first free channel, then the genie instructs the CRs to select the sensing order s with probability γ and to select the sensing order j with probability $(\frac{1-\gamma}{N-1})$, $\forall j \in \mathbf{S}, j \neq s$; otherwise, it instructs the CRs to again use the sensing order s in the next time slot. If all M CRs select different sensing orders the genie notifies the CRs that they have arrived at collision-free sensing orders; otherwise, it instructs the CRs to use the sensing order s in the next time slot. The process is repeated until the CRs arrive at collision-free sensing orders. The genie-aided modification is described in Fig. 8.

The probability that the $M = N$ CRs can arrive at collision-free sensing orders in a time slot using the genie aided scheme is given by

$$P_{E,genie} = \left[\underbrace{\sum_{i=1}^{N} \{(1-\theta_i) \prod_{j=1}^{N} (1-P_{fa,j}) \prod_{k=1}^{i-1} \theta_k\}}_{\text{Probability that } M=N \text{ CRs collide in the first free channel}} \times \overbrace{(\gamma)(\frac{1-\gamma}{N-1})^{N-1}[N!]}^{\text{Probability that all CRs select different sensing orders}} \right] \quad (1)$$

Let us now consider our proposed γ-persistent strategy, without a genie. Using the γ-persistent strategy, initially, M autonomous CRs select independently and randomly (with equal probability) a row of a common pre-defined Latin Square, and use the selected row as a sensing order. Using the γ-persistent strategy, the probability that the $M = N$ autonomous CRs arrive at collision-free sensing orders in the initial time slot is given by

$$P_{E,1} = \left(\frac{1}{N}\right)^{M} \left[\frac{N!}{(N-M)!}\right] = \frac{N!}{N^N} \quad (2)$$

Using the γ-persistent strategy, the probability that the M CRs arrive at collision-free sensing orders in the initial time slot is greater than $P_{E,genie}$, i.e., $P_{E,1} > P_{E,genie}$ due to the following reason. Considering $\theta_i = 0$, $\forall i \in \mathbf{N}$, $P_{fa,j} = 0$, $\forall j \in \mathbf{M}$ and $N = M$, $P_{E,genie}$ is given by:

$$P_{E,genie} = \overbrace{(\gamma)(\frac{1-\gamma}{N-1})^{N-1}[N!]}^{\text{Probability that all CRs select different sensing orders}} \quad (3)$$

It is easy to see that in equation (3), $P_{E,genie}$ as a function of γ is a continuous function. By taking the first derivative of equation (3) with respect to γ we have:

$$\frac{d}{d\gamma}P_{E,genie} = \frac{N!}{(N-1)^{N-1}}\left[(1-\gamma)^{N-1} - \gamma(N-1)(1-\gamma)^{N-2}\right]$$

Solving $\frac{d}{d\gamma}P_{E,genie} = 0$, we get $\gamma = \frac{1}{N}$. We evaluate $P_{E,genie}$ (given in equation 3) at $\gamma = \frac{1}{N}$, 0 and 1, i.e., at the critical point and the boundary points. We get that $P_{E,genie}(1/N) = \frac{N!}{N^N}$ and $P_{E,genie}(0) = P_{E,genie}(1) = 0$. From this, we see that the absolute maximum is obtained at $\gamma = 1/N$. By substituting $\gamma = 1/N$ in equation (3), we get $P_{E,genie} = P_{E,1}$. It is easy to see that for $\theta_i > 0$, $P_{fa,j} > 0$ or both, $P_{E,1} > P_{E,genie}$.

The probability that $M = N$ CRs autonomously arrive at collision-free sensing orders in the later time slots is always greater than $P_{E,genie}$ due to the following reason. The genie-aided modification corresponds to a worst case situation in which M CRs can only select the sensing orders randomly if they all collide in the first free channel of the sensing order s, and after collision, in the next time slot they select the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

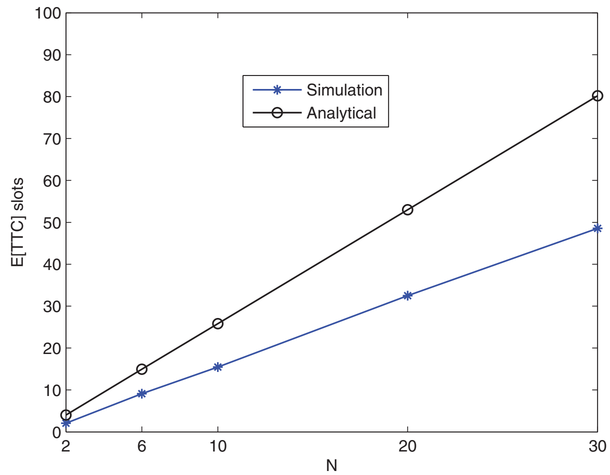


Fig. 9. Theoretical upper bound and simulated E[TTC] for different number of channels N and CRs M, where $N = M$.

sensing order s with probability γ and the sensing order j with probability $(\frac{1-\gamma}{N-1})$, $\forall j \in \mathbf{S}, \forall j \neq s$. In contrast, according to our proposed strategy when all CRs select the same sensing order, they may collide in any of the N sensing steps, and a colliding CR may select a sensing order either randomly with uniform probability or with probability γ (depending on the state of its binary flag and success counter). Moreover, for the γ-persistent strategy, any CR not experiencing a collision does not select a new sensing order from the Latin Square. Hence, the probability that each CR selects a different sensing order in any time slot is higher than the genie aided scheme, since the CR retains its previous sensing order.

It can be seen from equation (1) that for the genie-aided modification, in any time slot (before convergence) $P_{E,genie} \in (0,1)$, and as $n \rightarrow \infty$ this implies:

$$\lim_{n\rightarrow\infty} [1-(1-P_{E,genie})^n] = 1$$

Since in any time slot the probability for M CRs to arrive autonomously at collision-free sensing orders (P_E) is greater than $P_{E,genie}$, the network converges to collision-free sensing orders. ■

Next, we provide an upper bound on the expected convergence time of the γ-persistent strategy for the scenario where all $N = M$ channels are free from primary user activity and M autonomous CRs with $P_{fa} = 0$.

Proposition 4.1: Consider a system where all N channels are free from primary user activity and M autonomous CRs with $P_{fa} = 0$ select sensing orders from an $N \times N$ Latin Square, where $N = M$. Then using the γ-persistent strategy the expected number of time slots until convergence to collision-free sensing

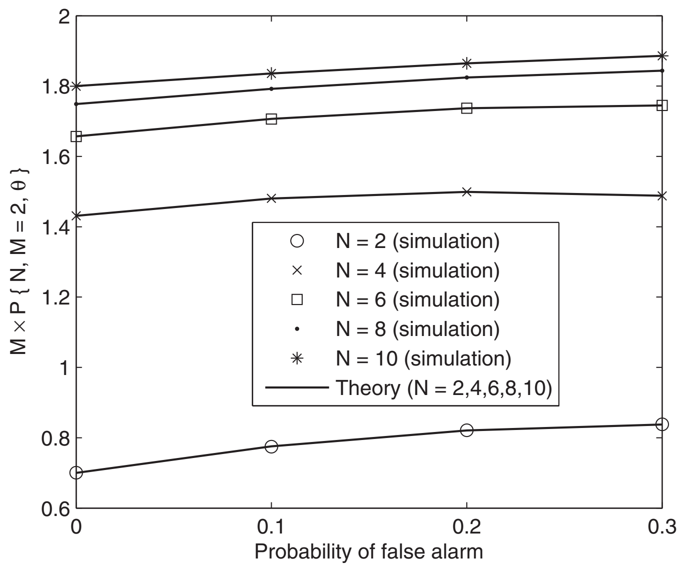


Fig. 10. Analytical and simulation results for the average number of successful transmissions per slot in the two-CR network ($M \times P\{N, M = 2, \theta\}$) as a function of false alarm probability for different number of channels N. $\theta_k = 0.3$, $\forall k \in \mathbf{N}$ and CRs employ a random sensing order selection strategy.

orders (E[TTC]) is $O(N)$.

Proof: For the scenario where $P_{fa} = 0$, it is easy to see that $\gamma = 1 - (\frac{1}{SC - log_2(P_{fa})})$ takes the value of $\gamma = 1$. This means that when a CR transmits successfully using a sensing order it persists with that sensing order with probability 1. The probability that a particular CR is the only one selecting a given sensing order is $((N-1)/N)^{N-1}$ and therefore the expected number of time slots until the CR is alone is $(N/(N-1))^{N-1}$. By linearity of expectation, the expected number of time slots for all CRs to arrive at collision-free sensing orders must be no more than $N^N/(N-1)^{N-1}$, and therefore E[TTC] is $O(N)$. ∎

In Fig. 9, we plot the theoretical upper bound (presented in Proposition 4.1) and simulated E[TTC] as a function of N. Unfortunately, the primary user activity in the channels and $P_{fa} > 0$ may slow down the convergence process (see Figs. 6 and 7). However for $0 < \theta_i < 1$, through extensive simulations we find that E[TTC] is increased for the γ-persistent strategy but is still $O(N)$.

V. Performance analysis and comparison with other strategies

A random channel sensing order scheme for distributed CR networks is proposed in [5]. In this section, we compare the performance of the proposed adaptive scheme, in terms of E[TTC] and the average number of successful transmissions in the network, against: 1) Random sensing order selection strategies (RPS and LS) [5]; and 2) a *randomize after every collision* (rand) adaptive strategy [21]. These two sensing order selection strategies can be explained as follows.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

$$\begin{aligned}
P(S) = \sum_{k=1}^{N} [P(& \text{ In the first } (k-1) \text{ steps, the channels visited by CR } i \text{ are either occupied by a PU or CR } i \text{ has} \\
& \text{false alarmed, and in the } k\text{th step CR } i \text{ visits a PU-free channel and does not collide with competing CR } j) \\
& + P(\text{ In the first } (k-1) \text{ steps, the channels visited by CR } i \text{ are either occupied or CR } i \text{ has} \\
& \text{false alarmed and in the } k\text{th step CR } j \text{ has given up channel search, i.e., it has found a PU-free channel} \\
& \text{and CR } i \text{ has false alarmed, and in the } (k+1)\text{th step CR } i \text{ finds a PU-free channel.})] \\
& = \sum_{k=1}^{N} \left\{ (1-\theta_k)(1-P_{fa,i})P_{fa,j} \prod_{l=1}^{k-1} \left(\theta_l + (1-\theta_l)P_{fa,i}P_{fa,j} \right) \right\} \\
& + \sum_{k=1}^{N-1} \left[(1-\theta_k)(1-P_{fa,j})P_{fa,i} \prod_{l=1}^{k-1} \left(\theta_l + (1-\theta_l)P_{fa,i}P_{fa,j} \right) \times \sum_{(m=k+1)}^{N} \left\{ (1-\theta_m)(1-P_{fa,i}) \prod_{(n=k+1)}^{m-1} (\theta_n + (1-\theta_n)P_{fa,i}) \right\} \right]
\end{aligned} \tag{5}$$

$$\begin{aligned}
P(D) = & \frac{1}{(N-1)} \Bigg[\sum_{k=1}^{N} \{ P(\text{In the first } (k-1) \text{ steps, the channels visited by CR } i \text{ are either occupied or CR } i \text{ has} \\
& \text{false alarmed and CR } j \text{ has given up channel search, i.e., it has already found a PU-free channel,} \\
& \text{and in the } k\text{th step CR } i \text{ visits a PU-free channel.}) + P(\text{In the first } (k-1) \text{ steps,} \\
& \text{the channels visited by CR } i \text{ are either occupied by a PU or CR } i \text{ has false alarmed and CR } j \text{ has not yet} \\
& \text{found a PU-free channel in one of these steps, and in the } k\text{th step CR } i \text{ visits a PU-free channel.}) \\
& + P(\text{ In the first } (k-1) \text{ steps, the channels visited by CR } i \text{ are either occupied by a PU or CR } i \text{ has false alarmed,} \\
& \text{and in the } k\text{th step, CR } i \text{ finds a PU-free channel (in these steps the probability of success of CR } i \text{ is not} \\
& \text{affected by the competing CR j))} \} \Bigg] \\
= & \frac{1}{(N-1)} \Bigg[\sum_{k=3}^{N} \Bigg\{ \sum_{l=k}^{N} (1-\theta_l)(1-P_{fa,i}) \sum_{(m=k-1)}^{l-1} \Bigg((1-\theta_m)(1-P_{fa,j}) \prod_{n=2}^{k-2} (\theta_n + (1-\theta_n)P_{fa,i}) \\
& \times \prod_{(o=k-1)}^{m-1} (\theta_o + (1-\theta_o)P_{fa,i}P_{fa,j}) \prod_{p=m+1}^{l-1} (\theta_p + (1-\theta_p)P_{fa,i}) \Bigg) \Bigg\} \Big(\theta_1 + (1-\theta_1)P_{fa,i} \Big) \\
& + \sum_{k=2}^{N} \Bigg\{ \sum_{l=k}^{N} \Bigg((1-\theta_l)(1-P_{fa,i})P_{fa,j} \prod_{m=1}^{k-1} (\theta_m + (1-\theta_m)P_{fa,i}) \prod_{n=k}^{l-1} (\theta_n + (1-\theta_n)P_{fa,i}P_{fa,j}) \Bigg) \Bigg\} \\
& + \sum_{k=1}^{N-1} \Bigg\{ \sum_{l=1}^{k} \Bigg((1-\theta_l)(1-P_{fa,i}) \prod_{m=1}^{l-1} (\theta_m + (1-\theta_m)P_{fa,i}) \Bigg) \Bigg\} \Bigg]
\end{aligned} \tag{6}$$

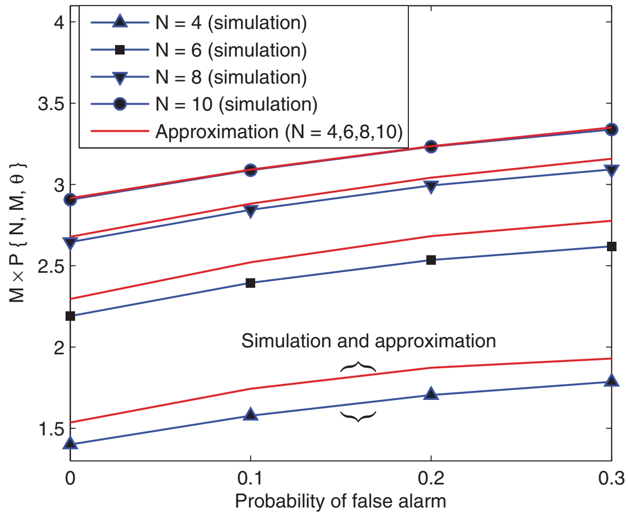
$$P\{N,M,\theta\} \approx \left(\frac{N-1}{N}\right)^{M-1} \sum_{k=1}^{Y+1} \left[(1-\theta_k)(1-P_{fa,i}) \prod_{j=1}^{k-1} \left(\theta_j + (1-\theta_j)P_{fa,i} \right) \right] + (\frac{M-1}{N})(\frac{N-1}{N})^{M-2} P(S) \tag{7}$$

where $P(S)$is given by equation (5) and $Y = \lceil (N-1)[1-\frac{1}{(N-1)}]^{M-1} \rceil$, $(N-1)[1-\frac{1}{(N-1)}]^{M-1}$ is the expected number of sensing orders that are not selected by any CR in a given time slot.

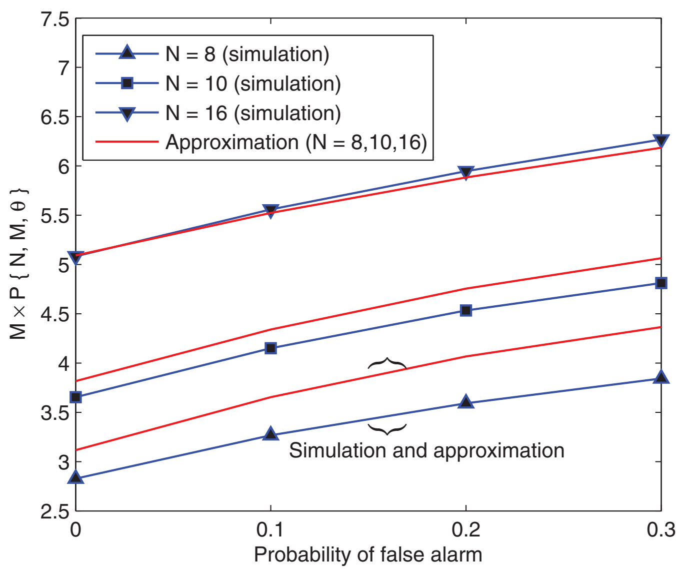
A. *Random sensing order selection strategy*

In each time slot, a sensing order (which may come from either the space of all permutations of N channels, or from a Latin Square) is independently and randomly selected (with equal probability) by each CR and then channels are sensed by each CR according to its sensing order.

Analysis of the random sensing order selection strategy:



(a) $M = 4$ CRs



(b) $M = 8$ CRs

Fig. 11. Analytical and simulation results for the average number of successful transmissions per slot in the M CR network $(M \times P\{N,M,\theta\})$ as a function of false alarm probability for different number of channels N. $\theta_k = 0.3$, $\forall k \in \mathbf{N}$ and CRs employ a random sensing order selection strategy.

We evaluate the performance of the random sensing order selection strategy (using a Latin Square) in terms of the average number of successful transmissions in the distributed CR network. For a given number of channels N there can be many Latin Squares. For instance the number of all possible Latin Squares for $N = 6$ is 812851200. We derive a closed form expression for the probability of success (see equations 4, 5 and 6) using a circulant matrix, which is an example of a Latin Square [10]. We compare the results obtained via derived closed form expression to results obtained via simulations in terms of average number of successful transmissions in the network (see Fig. 10). The simulation results are obtained when

CRs employ a common predefined Latin Square that is randomly selected out of many Latin Squares. It can be seen that the values calculated from Monte-Carlo simulations agree perfectly with those obtained from closed-form equations, i.e., the average number of successful transmissions in the network is not affected by the choice of a Latin Square.

An exact closed-form expression for the probability of success can be derived for any N when $M = 2$. For $M > 2$, obtaining an exact closed-form expression for the probability of success is challenging due to the combinatorial explosion in the number of ways that M CRs can find channels free from PUs and other CRs. We provide an approximation for the probability of success for any N and $M > 2$.

a) For $M = 2$ CRs and for any N, an exact closed-form expression for the probability of success for an individual CR i (assuming i.i.d PU occupancy model with $\theta_k = \theta,\ \forall\ k \in \mathbf{N}$) can be obtained as

$$P\{N, M = 2, \theta\} = \frac{1}{N} P(S) + \left(\frac{N-1}{N}\right) P(D) \tag{4}$$

where $P(S)$ represents the success probability of an individual CR given that the two CRs select the same sensing order, and $P(D)$ represents the success probability of an individual CR given that the two CRs select different sensing orders. The closed-form expressions for $P(S)$ and $P(D)$ are given in (5), (6). The derivations of equation (5) and (6) are given in the appendix.

In Fig. 10, we plot the average number of successful transmissions in a given time slot in the network as a function of the false alarm probability, for $M = 2$ and different values of N. We compare the results given by the closed-form expression we derived in (4) and the calculated average number of successful transmissions from a Monte Carlo simulation. Observe that the values calculated from Monte-Carlo simulations agree perfectly with those obtained from equation (4).

b) For $M > 2$ CRs, the difficulty in deriving an exact closed-form expression is that, in any sensing step k, the number of other competing CRs depends on how many CRs were successful in previous steps (which in turn determines how many channels are available) and how many have collided. Hence, instead of presenting an exact closed-form expression we present an approximation for the probability of success for an individual CR for any N and $M > 2$. The approximation is given in (7). The derivation of equation (7) is given in the appendix.

In Figs. 11a and 11b, we plot the average number of successful transmissions in a given time slot in the network as a function of the false alarm probability, for $M = 4, 8$ and different values of N. We compare the results given by the approximation we derived in (7) and the calculated average number of

TABLE II
OPTIMAL FALSE ALARM PROBABILITY FOR DIFFERENT VALUES OF $\theta_1 = \theta_2$ WHEN $N = M = 2$.

$\theta_1 = \theta_2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
P^*_{fa}	0.2884	0.2960	0.3025	0.3081	0.3131	0.3174	0.3213	0.3248	0.3279	0.3307

successful transmissions from a Monte Carlo simulation. Observe that the values calculated from Monte-Carlo simulations are within $\pm 12\%$ of those obtained from equation (7) and the approximation improves for larger values of N.

As shown in Fig. 6b, when adaptive strategies are employed for sensing order selection, sensing observations with false alarms lead to slower convergence to collision-free sensing orders, which in turn decreases the average number of successful transmissions in the network. However, when no adaptation is employed, i.e., when CRs use a random sensing order selection strategy, sensing observations with some false alarm increases the average number of successful transmissions in the network, as compared to when sensing observations do not generate false alarms (see Figs. 10, 11a and 11b). This counter-intuitive result stems from reduced collisions among the autonomous CRs and is further explained as follows. When sensing observations do not generate false alarms, the probability of success for two or more CRs that select the same sensing order is zero, as two or more autonomous CRs that select the same sensing order will find the same channel free (if one exists), transmit in that channel and collision will occur. However, if two or more CRs (with non-zero probabilities of false alarms) select the same sensing order then a CR can be successful in finding a free channel if one of the CRs does not generate a false alarm in a PU-free channel and the other CRs generate false alarms in that channel. Note that this gain in the probability of success of a CR (due to the non-zero probability of false alarm) is only valid for some values of false alarm probability, as for high values of false alarm probability, the likelihood of finding a free channel for the CR is reduced. We formalize this observation, for the case of two channels and two CRs, in the claim below.

Claim 5.1: Under the random sensing order selection strategy, when two autonomous CRs search two frequency channels sequentially with detection probabilities equal to 1, the success probabilities of both CRs are maximized at values of false alarm probabilities that are strictly greater than zero.

Proof: Let θ_1 and θ_2 be the probabilities of a PU being present in the two channels, where $\theta_1, \theta_2 < 1$, and $P_{fa,1} = P_{fa,2} = P_{fa}$ be the false alarm probabilities of the two CRs. Note that for the case where $P_{fa,1} \neq P_{fa,2}$, it is trivial to show that the success probability of an individual CR is maximized when

TABLE III
E[TTC] SLOTS FOR DIFFERENT ADAPTIVE STRATEGIES. THE NUMBER OF CHANNELS $N = 10$, FALSE ALARM PROBABILITY OF EACH CR IS SET TO 0.2 AND THE PROBABILITY OF THE PU BEING PRESENT (I.I.D MODEL) IS $\theta_i = 0.3$, $\forall i \in \mathbf{N}$.

	$M = 4$ CRs	$M = 6$ CRs	$M = 10$ CRs
rand	2.3207	5.8555	499.0901
$\gamma-persistent, \gamma = 0.8$	2.6747	5.9249	51.0014
$\gamma-persistent, \gamma = 0.5$	2.3481	5.2689	100.2261

its false alarm probability is zero and the false alarm probability of the other CR is one. However, we consider the case where the success probabilities of both CRs are maximized. Using (5) and (6), the success probability of an individual CR is

$$P\{N=2, M=2, (\theta_1, \theta_2)\} = \frac{1}{4}\Big[(2-\theta_1-\theta_2)(1-P_{fa}^2) + 2\{\theta_2(1-\theta_1)+\theta_1(1-\theta_2)\}(1-P_{fa})P_{fa} + 2(1-\theta_1)(1-\theta_2)\{(1-P_{fa}^2)P_{fa}^2 + P_{fa}(1-P_{fa})^2\}\Big] \tag{8}$$

Equation (8) is twice-differentiable, because it is a polynomial. By taking the first and second derivatives of (8) with respect to P_{fa}, we have

$$\frac{d}{dP_{fa}}P = \frac{1}{4}\Big[(2-\theta_1-\theta_2)(-2P_{fa}) + 2\{\theta_2(1-\theta_1)+\theta_1(1-\theta_2)\}(1-2P_{fa}) + 2(1-\theta_1)(1-\theta_2)\{1-2P_{fa}+3P_{fa}^2-4P_{fa}^3\}\Big] \tag{9}$$

and

$$\frac{d^2}{dP_{fa}^2}P = \frac{1}{4}\Big[-4-2\theta_1-2\theta_2+8\theta_1\theta_2+2(1-\theta_1)(1-\theta_2)\{-2+6P_{fa}-12P_{fa}^2\}\Big] < 0,\ \forall P_{fa} \tag{10}$$

so P is strictly concave $\forall \theta_1, \theta_2 < 1$. The optimal value holds when $\frac{d}{dP_{fa}}P = 0$. Solving $\frac{d}{dP_{fa}}P = 0$, we get $P_{fa}^* > 0$ (see Table II), $\forall \theta_1, \theta_2 < 1$. Hence the success probabilities of both CRs are maximized at values of false alarm probabilities that are strictly greater than zero. ■

B. Randomize after every collision (rand) strategy

In this subsection, we analyze a *randomize after every collision* (rand) strategy that utilizes adaptive randomization based on feedback for the CRs to arrive at collision-free sensing orders. In this strategy, initially each CR independently and randomly (with equal probability) selects a sensing order (which may come from either the space of all permutations of N channels, or from a Latin Square). In the next time slots, a CR randomly (with equal probability) selects a new sensing order only if it has experienced a collision in the previous slot; otherwise, it retains the previously selected sensing order. The basic idea is

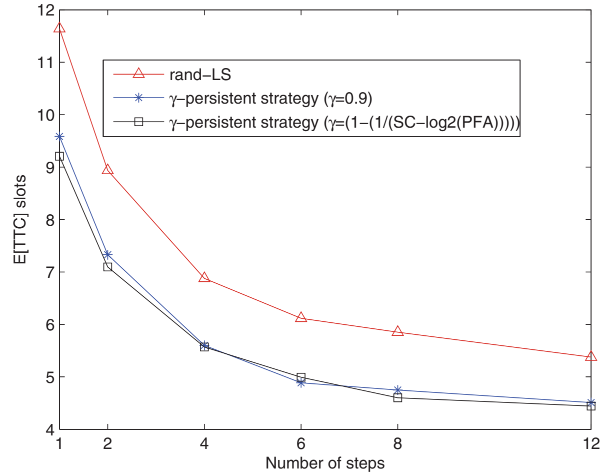


Fig. 12. E[TTC] as a function of the number of sensing steps. The number of channels N is 16, the number of CRs M is 8, false alarm probability of each CR is set to 0.2 and the probability of the PU being present in a given time slot (i.i.d model) is $\theta_i = 0.7$, $\forall i \in \mathbf{N}$.

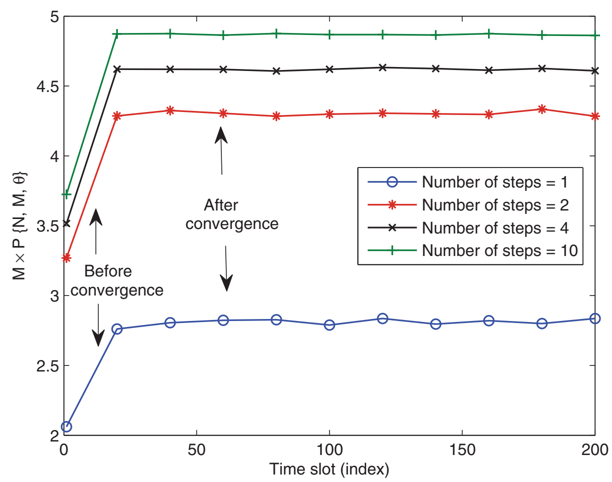


Fig. 13. Simulation results for the average number of successful transmissions for the γ-persistent strategy ($\gamma = 1 - (\frac{1}{SC - log_2(P_{fa})})$) in a given time slot for different strategies when $\theta_i = 0.3$, $\forall i \in \mathbf{N}$, false alarm probability of each CR is set to 0.2, $N = 10$, and $M = 5$.

for CRs to randomize their sensing orders in a way that leads them to distributedly arrive at collision-free sensing orders.

In Table III, we compare the randomize after every collision (rand) strategy with our proposed γ-persistent strategy in terms of E[TTC]; in both cases, selected sensing sequences come from a Latin Square (LS). Table III shows that the proposed γ-persistent strategy performs at least as well or significantly better than the rand strategy.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

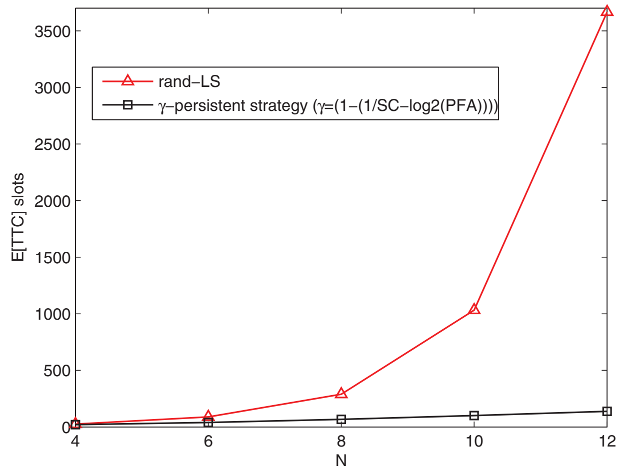


Fig. 14. E[TTC] as a function of varying the number of channels N for the scenarios where $N = M$, $\theta_i = \theta$=0.5, $\forall i \in \mathbf{N}$, false alarm probability of each CR set to 0.2 and number of sensing steps= 1.

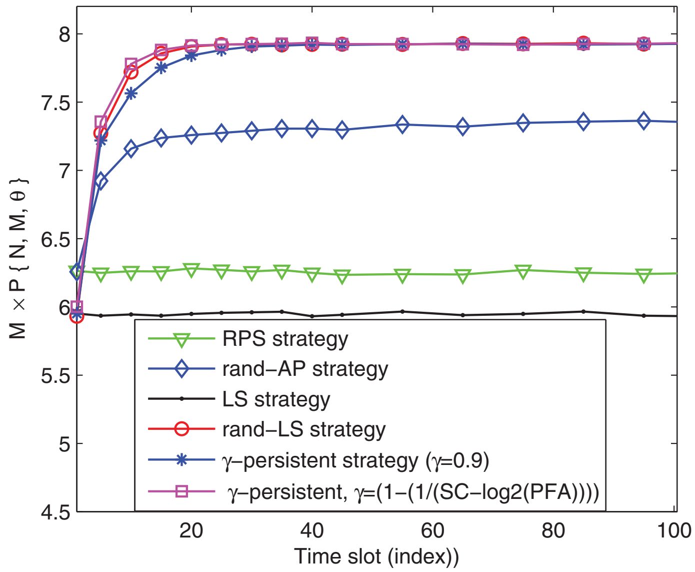
C. Simulation Results

We conduct simulations to compare the performance of the proposed γ-persistent strategy with other sensing order selection strategies in terms of E[TTC] and the average number of successful transmissions in a given time slot for different scenarios.

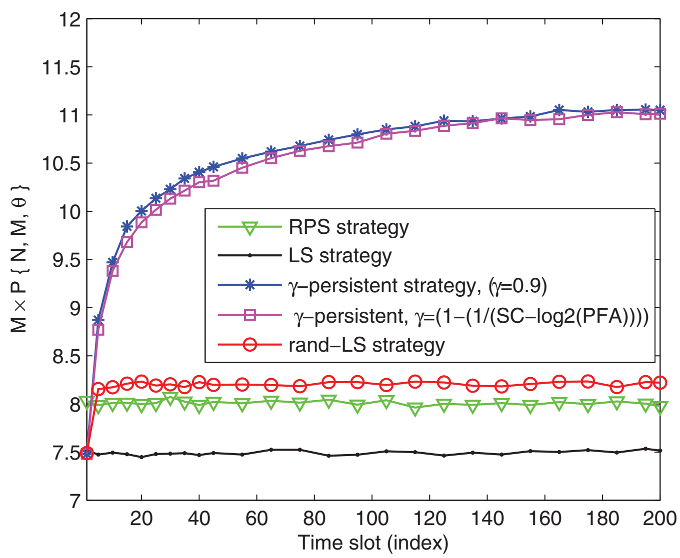
Figs. 12 and 13 evaluate the effect of varying the number of sensing steps on the performance of the proposed scheme in terms of E[TTC] and the average number of successful transmissions $(M \times P\{N, M, \theta\})$. All simulated scenarios in Fig. 12 consider the case $NT_{sensing} > T$, i.e, CRs cannot sense all N channels within the duration of a slot. In Fig. 13, except from one scenario where number of steps is 10, all scenarios consider the case $NT_{sensing} > T$. It can be seen in Figs. 12 and 13 that E[TTC] decreases and the average number of successful transmissions increases as the number of sensing steps are increased. However, it can also be seen in Figs. 12 and 13 that for the considered scenarios there is little or no gain when more than 4 sensing steps are utilized by an autonomous CR.

In Fig. 14 we evaluate our proposed strategy for the scenario where autonomous CRs are able to sense only one channel in a given time slot. It can be seen that our proposed adaptive persistent strategy significantly outperforms the randomize after every collision (rand) adaptive strategy.

In Figs. 15a and 15b, for $N = 16$ channels, we compare the average number of successful transmissions achieved by the different strategies under two different scenarios, when $M < N$ and when $M = N$. From the two figures we can see that the adaptive sensing order selection strategies achieve the highest average number of successful transmissions. The two figures (Figs. 15a and 15b) show that the proposed γ-



(a) $N = 16$ channels and $M = 8$ CRs



(b) $N = 16$ channels and $M = 16$ CRs

Fig. 15. Simulation results for the average number of successful transmissions in a given time slot $(M \times P\{N,M,\theta\})$ for different strategies when $\theta_k = 0.3$, $\forall k \in \mathbf{N}$ and false alarm probability of each CR set to 0.2.

persistent strategy performs at least as well or significantly better than all other sensing order selection strategies evaluated. Particularly for the scenario where $M = N$, the γ-*persistent* strategy significantly outperforms all the other strategies evaluated (see Fig. 15b). The two figures (Figs. 15a and 15b) also compare the performance of the random sensing order selection strategy for two different scenarios, when selected sensing orders come from the space of all permutations of N channels (RPS strategy), and when selected orders come from a Latin Square (LS strategy). It can be seen from the two figures (Figs. 15a and 15b) that the average number of successful transmissions is greater for the RPS strategy than for the

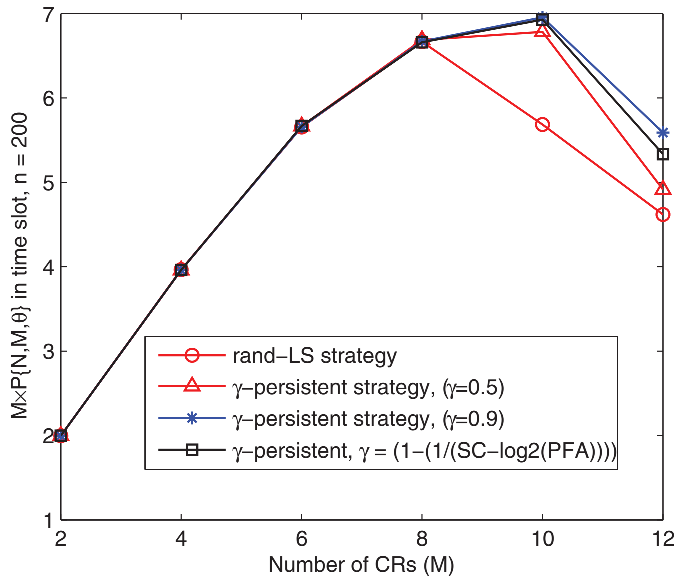


Fig. 16. Simulation results for the average number of successful transmissions achieved in time slot, $n = 200$, as a function of number of CRs for different strategies when $\theta_k = 0.3$, $\forall k \in \mathbf{N}$ and false alarm probability of each CR set to 0.2. The number of channels N is 10. Calculations are performed by Monte Carlo method using 30000 Monte Carlo runs for opportunistic transmissions in 200 time slots using different strategies.

LS strategy. This is due to the fact that the RPS strategy increases the number of ways to visit N channels and therefore reduces the chances of collisions between CRs. However, when adaptation is employed then the scenario where CRs select sensing orders from a common pre-defined Latin Square, i.e., randomize after every collision using a Latin Square (rand-LS) strategy, results in increased number of successful transmissions, as compared to the case where they select sensing orders from the space of all permutations of N channels, i.e., randomize after every collision using all permutations (rand-AP) strategy, (see Fig. 15a).

In Fig. 16, for $N = 10$ channels, we compare the average number of successful transmissions achieved in time slot, $n = 200$, as a function of number of CRs for different adaptive strategies. It can be seen from Fig. 16 that when $N = 10$ channels are available for opportunistic transmissions then for difficult scenarios, i.e., $N \leq M$, the proposed γ-persistent strategy maximizes the average number of successful transmissions as compared to the *randomize after every collision* (rand-LS) strategy; in all cases, selected sensing orders come from a Latin Square.

When $N < M$ it is not possible for all CRs to select collision-free sensing orders. However, Fig. 16 shows that for $N < M$ our proposed strategy enables the CRs to reduce the likelihood of collisions. Fig. 16 also shows that, for the proposed γ-persistent strategy, when $N < M$, high values of γ, i.e., $\gamma = 0.9$ (high persistence) performs slightly better, as compared to when γ is employed as a function of false

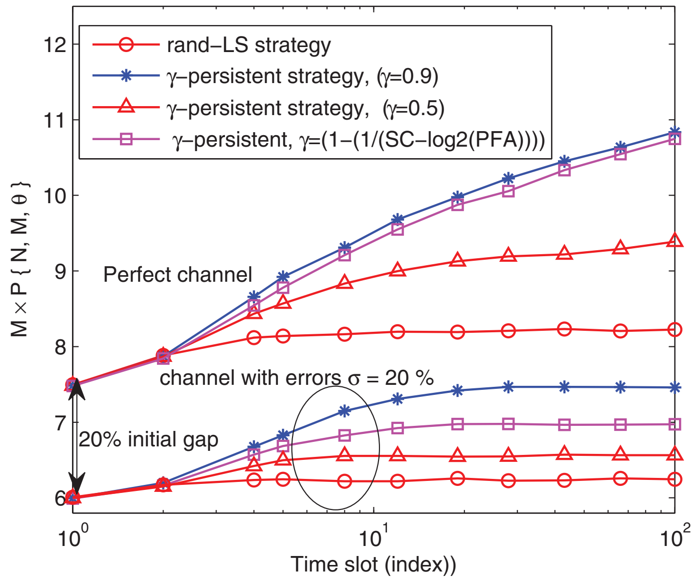


Fig. 17. Simulation results for the average number of successful transmissions in a given time slot $(M \times P\{N,M,\theta\})$ for different strategies when $\theta_k = 0.3$, $\forall k \in \mathbf{N}$ and false alarm probability of each CR is set to 0.2. The number of channels N is 16 and the number of CRs M is 16.

alarm probability and SC. This is due to the fact that for $N < M$ there are more chances of collisions and high persistence enables successful CRs to stick with their sensing orders with high probability.

The presented adaptive sensing order selection strategies use transmission failure as an indication that a collision has occurred, i.e., the selected sensing order is also used by another CR. In practise, transmission failures can also be due to channel errors introduced by fading. In Fig. 17, we compare the performance of the adaptive sensing order selection strategies with and without channel errors. We use a simple model where channel errors are introduced at a particular rate σ. It can be seen in Fig. 17 that the performance of the proposed adaptive strategies is degraded due to channel errors ($\sigma = 20\%$). However, it can also be seen in Fig. 17 that for different values of γ, the proposed γ-persistent strategy outperforms the *randomize after every collision* (rand-LS) adaptive strategy; in all cases, selected sensing orders come from a Latin Square. We note in Fig. 17 that high persistence $\gamma = 0.9$ is more robust to the presence of channel errors. This is due to the fact that high persistence allows a successful CR to stick with its sensing order (after experiencing a collision) with high probability even after few consecutive transmission failures.

VI. Conclusions and Future Work

In this research we design an efficient adaptive persistent sensing order selection strategy for a distributed cognitive radio (CR) network. We find that the performance that results from a random selection of sensing orders is limited by the collisions among the autonomous CRs. We propose a γ-*persistent* strategy that

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

enables the CRs to reduce the likelihood of collisions with one another. We show that the γ-*persistent* strategy converges to collision-free channel sensing orders. We also find that when adaptation is employed, there is an increase in the average number of successful transmissions in the network when the CRs select sensing orders from a predefined Latin Square, as compared to when they select sensing orders from the space of all permutations of N channels. We also explore the effects of false alarms and channel errors on adaptation decisions. We show that even in the presence of false alarms and channel errors, the proposed γ-*persistent* strategy increases the average number of successful transmissions in the network.

Our model considers competition for a set of channels to be occupied opportunistically by a set of frequency-agile radios. It is not necessarily restricted to a connected network, but in our derivations we assume a zero probability of missed detection of a primary user. One of the extensions we envision for the model is to consider the case when the set of available channels is non-homogeneous across the cognitive nodes, representing a network covering a wider geographic area. This scenario is part of our ongoing and future work. Moreover, we will also extend this research to explore the impact of asymmetric interference links, i.e., when CR i interferes with CR j but not vice-versa, on adaptation decisions.

REFERENCES

[1] S. Huang, X. Liu, and Z. Ding, "Opportunistic spectrum access in cognitive radio networks," in *Proceedings of the IEEE International Conference on Computer Communications (INFOCOM)*, Phoenix, AZ, USA, Apr. 2008, pp. 1427–1435.

[2] A. Ghasemi and E. S. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Communications Magazine*, vol. 46, no. 4, pp. 32–39, Apr. 2008.

[3] R. Fan and H. Jiang, "Optimal multi-channel cooperative sensing in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 9, no. 3, pp. 1128–1138, Mar. 2010.

[4] ——, "Channel sensing-order setting in cognitive radio networks: a two-user case," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 9, pp. 4997–5008, Nov. 2009.

[5] H. T. Cheng and W. Zhuang, "Simple channel sensing order in cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 676–688, 2011.

[6] A. Anandkumar, N. Michael, and A. Tang, "Opportunistic spectrum access with multiple users: Learning under competition," in *Proceedings of the IEEE International Conference on Computer Communications (INFOCOM)*, 2010, pp. 1–9.

[7] N. B. Chang and M. Liu, "Competitive analysis of opportunistic spectrum access strategies," in *Proceedings of the IEEE International Conference on Computer Communications (INFOCOM)*, Apr. 2008, pp. 1535–1542.

[8] K. Liu and Q. Zhao, "Distributed learning in multi-armed bandit with multiple players," *IEEE Transactions on Signal Processing*, vol. 58, no. 11, pp. 5667–5681, Nov. 2010.

[9] C. F. Laywine and G. L. Mullen, *Discrete Mathematics using Latin Squares*, 1st ed., ser. Wiley-Interscience Series in Discrete Mathematics and Optimization. New York: John Wiley & Sons, 1998.

[10] P. J. Davis, *Circulant Matrices*, 2nd ed. Chelsea Publishing Company, 1994.

[11] Q. Zhao, S. Geirhofer, L. Tong, and B. M. Sadler, "Opportunistic spectrum access via periodic channel sensing," *IEEE Transactions on Signal Processing*, vol. 56, no. 2, pp. 785–796, Feb. 2008.

[12] Y. C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1326–1337, 2008.

[13] N. C. Theis, R. W. Thomas, and L. A. DaSilva, "Rendezvous for cognitive radios," *IEEE Transactions on Mobile Computing*, vol. 10, no. 2, pp. 216–227, 2011.

[14] A. Anandkumar, N. Michael, A. K. Tang, and A. Swami, "Distributed algorithms for learning and cognitive medium access with logarithmic regret," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 731–745, 2011.

[15] H. Li, "Multi-agent Q-learning for Aloha-like spectrum access in cognitive radio systems," *EURASIP Journal on Wireless Communications and Networking*, vol. 2010, pp. 1–15, Apr. 2010.

[16] K. Liu and Q. Zhao, "Distributed learning in cognitive radio networks: Multi-armed bandit with distributed multiple players," in *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, 2010, pp. 3010–3013.

[17] Y. Gai, B. Krishnamachari, and R. Jain, "Learning multiuser channel allocations in cognitive radio networks: A combinatorial multi-armed bandit formulation," in *Proceedings of the IEEE International Dynamic Spectrum Access Networks (DySPAN)*, 2010, pp. 1–9.

[18] H. Liu, B. Krishnamachari, and Q. Zhao, "Cooperation and learning in multiuser opportunistic spectrum access," in *Proceedings of the IEEE International Conference on Communications Workshops (ICC workshops)*, 2008, pp. 487–492.

[19] H. Jiang, L. Lai, R. Fan, and H. V. Poor, "Optimal selection of channel sensing order in cognitive radio," *IEEE Transactions on Wireless Communications*, vol. 8, no. 1, pp. 297–307, Jan. 2009.

[20] N. B. Chang and M. Liu, "Optimal channel probing and transmission scheduling for opportunistic spectrum access," in *Proceedings of the Second Workshop on Information Theory and Applications*, 2007, pp. 27–38.

[21] Z. Khan, J. Lehtomaki, M. Mustonen, and M. Matinmikko, "Sensing order dispersion for autonomous cognitive radios," in *Proceedings of the 6th International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*, 2011, pp. 1–5.

[22] J. Barcelo, B. Bellalta, C. Cano, A. Sfairopoulou, and M. Oliver, "Carrier sense multiple access with enhanced collision avoidance: a performance analysis," in *Proceedings of the International Conference on Wireless Communications and Mobile Computing (IWCMC)*, Jun. 2009, pp. 733–738.

[23] J. Barceló, B. Bellalta, C. Cano, and M. Oliver, "Learning-BEB: Avoiding collisions in WLAN," in *Proceedings of the EUNICE Summer School*, 2008, pp. 1–8.

[24] M. Fang, D. Malone, K. R. Duffy, and D. J. Leith, "Decentralised learning MACs for collision-free access in WLANs," Sep. 2010. [Online]. Available: http://arxiv.org/abs/1009.4386v1

[25] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proceedings of the IEEE International Dynamic Spectrum Access Networks (DySPAN)*, 2005, pp. 131–136.

[26] E. Peh and Y.-C. Liang, "Optimization for cooperative sensing in cognitive radio networks," in *Proceedings of the IEEE International Wireless Communication Networking Conference (WCNC)*, 2007, pp. 27–32.

[27] H. Su and X. Zhang, "Cross-layer based opportunistic MAC protocols for QoS provisionings over cognitive radio wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 26, pp. 118–129, 2008.

PLACE PHOTO HERE

Zaheer Khan Zaheer Khan received his M.Sc. in Electrical engineering from the University College Borås, Sweden, in 2007. Since August 2008, he is a Ph.D. candidate at the Centre for Wireless Communications (CWC), University of Oulu, Oulu, Finland. He is the recipient of the Marie Curie fellowship for the year 2007-08. During the summer of 2010, he worked as a visiting researcher at Telecommunications Research Centre (CTVR), Trinity College Dublin, Ireland. Zaheer's research interests include application of game theory to model distributed wireless networks, cognitive and cooperative communications, and wireless signal design. Specifically, he has been working on the design of distributed and adaptive resource allocation strategies for cognitive radio networks.

PLACE PHOTO HERE

Janne J. Lehtomäki Janne Lehtomäki received the M.Sc. and Dr.Sc. degrees in Telecommunications from University of Oulu in 1999 and 2005, respectively. Currently, he is a postdoctoral research fellow at the Centre for Wireless Communication (CWC), University of Oulu, Oulu, Finland. In 2008, he worked as a visiting researcher at Tokyo University of Agriculture and Technology. His research interests are in statistical signal processing and cognitive radios. He is a member of IEEE.

PLACE PHOTO HERE

Luiz A. DaSilva Luiz A. DaSilva [SM] currently holds the Stokes Professorship in Telecommunications in the Department of Electronic and Electrical Engineering at Trinity College, Dublin. He has been on the faculty of Virginia Tech's Bradley Department of Electrical and Computer Engineering since 1998. Prof. DaSilva's research focuses on distributed and adaptive resource management in wireless networks, and in particular cognitive radio networks. Other areas of current interest include dynamic spectrum access and the application of game theory to modeling distributed wireless networks.

PLACE PHOTO HERE

Matti Latva-aho Matti Latva-aho [SM] received the M.Sc. (E.E.), Lic.Tech. and Dr. Tech degrees from the University of Oulu, Finland in 1992, 1996 and 1998, respectively. From 1992 to 1993, he was a Research Engineer at Nokia Mobile Phones in the CDMA systems research group. During the years 1994-1998 Matti was a Research Scientist at Telecommunication Laboratory and Centre for Wireless Communications (CWC) at the University of Oulu. After graduation he became Director of CWC in 1998 and head of Department of Communications Engineering in 2011.